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THE

PHYSICAL REVIEW.

ON THE ELECTRIC STRENGTH OF SOLID, LIQUID, AND GASEOUS DIELECTRICS.

By Alexander Macfarlane and G. W. Pierce.

IN the course of a research on "The Disruptive Discharge of Electricity,"[1] Dr. Macfarlane obtained the result that the electrostatic gradient necessary to force a spark through a thin stratum of dielectric diminishes as the thickness increases, when air or other gas is the dielectric; but remains constant when turpentine or other insulating liquid is the dielectric. Mr. Steinmetz has recently obtained the same result,[2] and in addition has shown that a solid dielectric, such as paraffined paper, behaves in the same manner as a liquid dielectric. In order to obtain further independent data, Mr. Pierce has made a series of measurements for paraffined paper, beeswaxed paper, and kerosene oil. We had not the means of making absolute measurements, but this was immaterial, as our object was to compare these solid and liquid dielectrics with air in the most direct manner possible. To effect the comparison two discharging tables were used. On each table a pair of brass discs, about four inches in diameter, were supported parallel to one another, and the connecting-rods were attached to the poles of a

[1] Trans. Roy. Soc. Edinb., Vol. XXVIII., p. 633, and Vol. XXIX., p. 561. Proc. R. S. E., Vol. X., p. 555; Phil. Mag., Dec., 1880; Trans. Amer. Inst. El. Eng., March, 1893.

[2] Trans. Amer. Inst. El. Eng., Feb., 1893.

Toepler-Holtz machine, so as to form two alternative paths for the discharge. One of the tables was provided with a micrometer-millimeter scale, by which the distance between the plates could be read off accurately. It was used for the air gap. The sheet, or sheets, of paraffined or beeswaxed paper was placed between the other pairs of discs, and the discs pressed together. The air interval was at the beginning made too small, so that the spark should pass through it; for the passage of a number of sparks does not alter the air stratum, while a single spark through the solid damages it permanently. The air stratum was gradually increased in thickness until the spark preferred to pass through

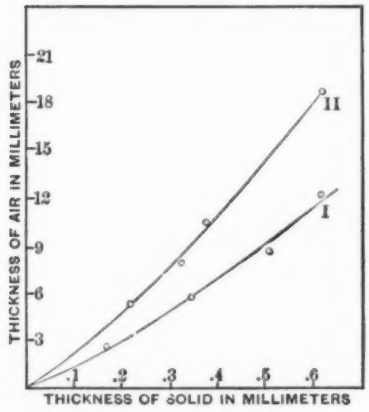

Diagram 1.

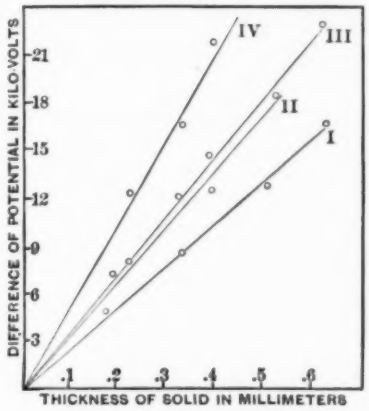

Diagram 2.

the solid. The thickness of the air interval was obtained by reading the micrometer; the equivalent thickness of the solid was ascertained by cutting out a small piece round the hole pierced by the discharge, and measuring the thickness by means of the spherometer. The sheets of paper used were prepared by taking sheets of linen typewriter paper, dipping them into the melted paraffin or beeswax, and drawing them out so as to make the saturation as uniform as possible. In the case of the kerosene oil, the plates were fixed horizontally in a glass vessel, the lower one supported on a conducting rod passing through the bottom of the vessel, the upper one suspended from a graduated rod which moved tight in the cover of the vessel. The oil was filtered before

use, and the bubbles of gas formed by a discharge removed before proceeding to another comparison.

Table I. appended gives the results for paraffined paper. The value given in the second column for the thickness of the paper, and that in the third column for the equivalent thickness of air, are the averages of all the determinations for the specified number of sheets. The fourth and fifth columns give the difference of potential in kilovolts required to pass a spark through the corresponding stratum of air, that of the fourth column being obtained from the papers of Dr. Macfarlane, while that in the fifth column is from the paper of Mr. Steinmetz.

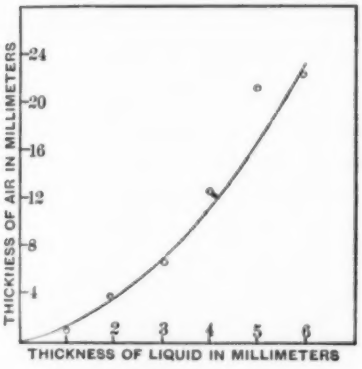

Diagram 3.

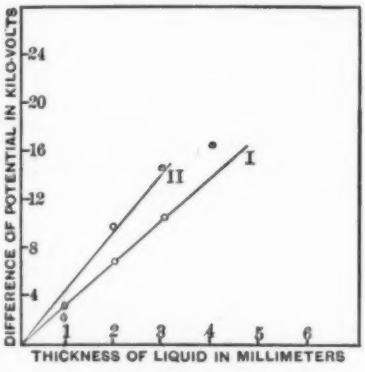

Diagram 4.

In Table II. the results for beeswaxed paper are tabulated in a similar manner.

Table III. gives the results for kerosene oil. The value entered for the equivalent stratum of air is the average of all the determinations made for the given stratum of oil.

The diagrams appended exhibit the results graphically. In Diagram 1, curve I. is for paraffined paper, and curve II. for beeswaxed paper; in Diagram 3 the curve is for kerosene oil.

The equivalent thickness of air is not proportional to the thickness of the solid or liquid, but increases more rapidly as the stratum increases. The curves are concave towards the axis of ordinates. Now the curve giving the differences of potential for different thicknesses of air is concave towards the axis of

abscissæ. Diagrams 2 and 4 are designed to test whether these opposite bendings neutralize one another. In Diagram 2, curve I. gives the difference of potential for paraffined paper, and curve III. for beeswaxed paper, using Steinmetz's values of the air equivalent; while curves II. and IV. give the same, using Macfarlane's values of the air-equivalent. In Diagram 4, curve I. is for Steinmetz's values, and curve II. for Macfarlane's values. All these curves are very approximately straight lines passing through the origin.

From these and the other results mentioned above we may conclude that thin strata of a solid or liquid dielectric are equally strong whatever the thickness, while thin strata of a gaseous dielectric grow weaker as the thickness is increased. Viewed in the light of the mathematical theory of electricity in equilibrium, it is the gaseous dielectric which is anomalous. The cause does not appear to be any surface phenomenon, but rather to be due to the greater rarity of a gas which allows discharge by convection to be more readily set up. For the greater thicknesses convection is sometimes started in a liquid dielectric, and it is observed that the discharge then takes place at a lower difference of potential.

A table of electrostatic gradients is appended, Table IV., which has been compiled from this and the other investigations mentioned. Where Steinmetz has a value for the same dielectric, it is placed in a parallel column.

TABLE I.

PARAFFINED PAPER.

Number of sheets.	Thickness of paper in millimeters.	Thickness of air in millimeters.	Diff. of Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	.174	2.63	7.2	5.
2	.339	5.42	12.6	8.9
3	.506	8.47	18.6	13.
4	.620	11.76	—	16.5

Table II.

BEESWAXED PAPER.

Number of sheets.	Thickness of paper in millimeters.	Thickness of air in millimeters.	Diff. in Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	.214	5.18	12.3	8.2
2	.326	8.05	16.7	12.1
3	.390	10.5	22.2	14.9
4	.606	18.3	—	22.3

Table III.

KEROSENE OIL.

Thickness of oil in millimeters.	Thickness of air in millimeters.	Diff. of Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	1.0	3.6	2.5
2	4.0	9.9	6.9
3	6.8	15.	10.5
4	12.5	—	17.
5	20.8	—	—

Table IV.

ELECTRIC STRENGTH.

Dielectric.	Electrostatic gradient in kilovolts per centimeter.		Dielectric.	Electrostatic gradients in kilovolts per centimeter.	
	Macfarlane.	Steinmetz.		Macfarlane.	Steinmetz.
Oil of turpentine	94	64	Paraffined paper .	360	339
Paraffin oil . .	87	—	Beeswaxed paper .	540	—
Olive oil . . .	82	—	Air (thickness, 5 mm.)	23.8	16
Paraffin (melted)	56	81	CO_2 " "	22.7	—
Kerosene oil. .	50	—	O " "	22.2	—
Paraffin (solid) .	130	—	H " "	15.1	—
			Coal gas	22.3	—

ON A METHOD OF PHOTOGRAPHING THE MANOMETRIC FLAME WITH APPLICATIONS TO THE STUDY OF THE VOWEL *A*.

BY ERNEST MERRITT.

THE manometric flame, with its attached capsule, has long been recognized as an important aid in the study of sound. Its many applications to the investigation of stationary waves, where the sensitiveness of the flame to small changes of pressure has made it of especial service, are too well known to require mention here. In the study of the quality of sounds the manometric flame has also been found to possess many advantages, since the extreme lightness of the moving parts makes it possible for them to follow the vibrations of complex sounds with almost perfect accuracy. The simplicity and elegance of the method which is thus afforded of detecting the presence of overtones have led to its wide use for purposes of demonstration, but the applications of the method have been almost wholly confined to experiments of an illustrative character. Koenig's investigation of the vowel sounds[1] is, I believe, the only example of the use of the manometric flame in the quantitative study of timbre.

The chief difficulty which has stood in the way of a more extensive use of the manometric flame undoubtedly lies in the transient character of the images that are seen in the revolving mirror. To obtain accurate drawings of these images for use in further study is so laborious a task that few are willing to undertake it, and the drawings by Koenig, which are found in all our text-books, will

[1] Rudolph Koenig, On Manometric Flames, Phil. Mag., Series iv., 45, p. 1; Pogg. Ann., cxlvi., p. 161. Also, Sur les notes fixes caractéristiques des diverses voyelles, Comptes Rendus, 70, p. 931.

probably long remain as monuments to the skill and patience of this great student of sound.

The possibility of photographing the manometric flame, and thus obtaining a permanent and accurate record of its vibrations, does not appear to have occurred to any of the earlier experimenters. The reason for this is doubtless to be found in the comparatively undeveloped state of the photographic art at that time. With the slow and cumbersome wet-plate processes of twenty years ago success in instantaneous photography could scarcely be expected. The improvements that have been made in photographic processes during recent years are, however, of a most radical nature, and many problems are now successfully attacked by photographic methods which but a few years ago would have been deemed incapable of solution. When the attention of the writer was called to the problem of photographing the manometric flame,[1] it therefore seemed to him not improbable that the use of modern dry-plates would lead to satisfactory results. It was with this hope that the work here described was begun.

Methods and Apparatus.

To obtain a photograph of the manometric flame which shall be available as a means of analysis requires (1) a lens by which an image of the flame may be thrown upon the sensitive plate, and (2) some device for producing a uniform movement of the image in a line at right angles to the direction of its vibration. This movement might be brought about by the use of a revolving mirror, or by a movement of the lens, either of which methods would doubtless have proven satisfactory. With the apparatus at hand, however, it appeared more convenient to keep both lens and image stationary, and to move the plate itself. The plate-holder, mounted in a suitable carriage, was therefore arranged to slide between guides across the back of the camera. With high speeds it was found necessary to extend these guides for about 50 cm.

[1] By Professor O. F. Emerson, who was desirous of making use of the manometric flame in connection with certain studies in phonetics. Although the results have proven satisfactory from the standpoint of the physicist, the method has been found to be unavailable for the purpose first considered.

on either side of the camera in order to avoid the jar of sudden starting and stopping. To obtain approximately the same speed in different experiments the carriage was usually set in motion by means of a rubber band, whose tension, when once adjusted, was left unaltered. To guard against stray light all parts of the apparatus were painted black, and the experiments were performed in a room which was partially darkened.

The difficulty from a photographic standpoint in obtaining good negatives of the manometric flame will be appreciated when it is remembered that the plate must move during each vibration through a distance at least as great as the width of the flame. If the motion is slower than this the vibrations will not be sharply separated, and the negative will be valueless so far as its use in analysis is concerned. For a sound whose pitch is 500 vibrations per second the exposure at any one point of the plate must therefore be less than .002 sec., while if wider separation is desired, or if the tone studied is of higher pitch, the time of exposure must be correspondingly less. It is not surprising therefore that all attempts to photograph manometric flames of the ordinary form proved unsuccessful. When gas was used which had been highly enriched with benzine and naphthalene it was indeed possible to obtain faint negatives, after a long and tedious development, and by the use of the most sensitive plates; but all attempts to make these negatives sufficiently dense for convenient measurement proved utterly fruitless. The use of large lenses and concave mirrors to increase the brightness of the image, and the employment of a variety of substances to enrich the gas, were alike without avail.

A sufficiently brilliant flame was finally obtained, however, by the adoption of a form of jet which enabled the gas to be burned in pure oxygen. I have since found that a somewhat similar method of producing an actinic flame has been used by M. Doumer,[1] who succeeded in photographing such flames in 1886, and afterwards applied his method[2] to the study of the French

[1] Mesure de la hauteur des sons par les flammes manométriques, Comptes Rendus, 103, p. 340.

[2] Des voyelles dont le caractère est très-aigu, Comptes Rendus, 105, p. 1247.

vowels *i* and *u*. Since, however, the description of M. Doumer's work appears to be confined to the brief notes in the Comptes Rendus which are referred to above, I have thought that a detailed description of the apparatus which I have found most serviceable, together with a presentation of some of the results obtained, may not be devoid of interest.

In Fig. 1 is shown a sectional diagram of the manometric capsule and jet which were finally employed. It will be seen that the burner differs from those commonly used only in the presence of the outer tube *T*, which surrounds the jet proper and extends as high as the bottom of the flame. Oxygen was admitted to this tube at *O*, while gas entered the capsule at *G* and proceeded through the short brass tubing *AA* to the flame *F*. The flame was thus surrounded by an atmosphere of oxygen, and was found to burn with a brilliant white light, strikingly different from that of the ordinary gas flame. By adjusting the pressure of the gas and oxygen, the height and brilliancy of the flame could be kept under perfect control. On account of the intense heat it was found necessary to make the tip of the burner of platinum, and the best results were obtained when the aperture at *B* was considerably larger than is customary in manometric jets. In the burner here described the opening at *B* was a little over 1 mm. in diameter.

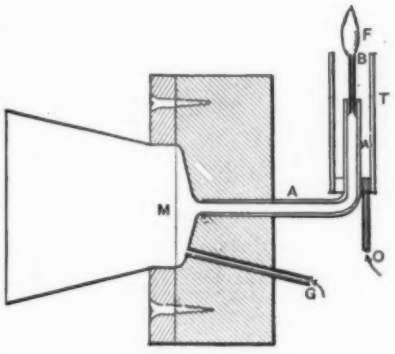

Fig. 1.

The manometric capsule, as will be seen in the figure, did not differ in any essential particular from those ordinarily used. Its size, however, was somewhat greater than is common, the diameter of the membrane being about 5 cm. The tubes leading to the jet were made as short as possible in order to reduce the effects of friction, and sudden changes in diameter were avoided. The membrane *M* was made of thin dentist's rubber. A metallic mouth-piece of the form shown served to concentrate the sound

waves upon the surface of the membrane, and so intensify the effect.

With the form of apparatus described above, the flame was found to be not only extremely brilliant, and of high actinic power, but also fully as sensitive to sound vibrations as manometric flames of the ordinary form. An adjustment of pressures which made the flame about 15 mm. high was found to give the best results. No difficulty was met with in obtaining sharp negatives of the vibrating flame with any of the ordinary instantaneous plates. By enriching the gas with benzine the actinic effect of the light was considerably increased, but with the plate moving at the rate of about two meters per second, which was the speed used throughout most of these experiments, the results obtained with ordinary gas were quite satisfactory. It should be mentioned that the camera was placed at such a distance from the jet as to make the image on the plate almost equal in size to the flame itself. The lens was used with full aperture, about 4 cm., and had a focal length of 40 cm.

In making use of the above apparatus for the study of vowel sounds, two flames were ordinarily used, one being placed vertically above the other. The upper flame was set in vibration by the sound to be studied, while the lower was connected with an organ pipe of known pitch. Both flames were photographed at the same time. A means of determining the pitch of the sound studied was thus afforded which was susceptible of a high degree of accuracy, the advantage of the method lying in the fact that its results were unaffected by a change in the speed of the moving plate. Prints from two of the negatives thus obtained are shown in Figs. 1 and 2 of Plate IV.

Results.

A few of the photographs that have so far been taken will be found reproduced in Figs. 3 to 8, Plate IV., and will serve to indicate the general character of the results obtained. Figs. 3 to 6 show the forms assumed by the manometric flame when set in vibration by the vowel sounds of *a* and *o* as they occur in the

words *father*, *law*, *hat*, and *know*. In each case the vowel has been sung at four different pitches, corresponding respectively to $C_2(128)$, E_2, G_2, and $C_3(256)$. In Fig. 7 the four vowels are represented upon the same plate for convenience in comparison, the pitch being 128 in each case. Fig. 8 shows the effect on the flame of the humming sound of *m*, also sung at four different pitches. It should be mentioned that the photographs shown in Figs. 3 to 8 were taken only to illustrate the results obtained. Examples of photographs actually used for measurement are shown in Figs. 1 and 2.

Although the flame was plainly visible to the eye throughout its entire length, the actinic effect appears to be confined to the upper portions. In some of the photographs a faint band of light may be detected which extends for some distance below the brighter portions of the image and indicates the position of the bottom of the flame. But in most cases it is only the light from the upper half of the flame that has left any trace upon the plate.

It will be seen that in the majority of cases an inspection of the photograph is sufficient to determine the pitch of the most prominent harmonic. In Fig. 1, for example, the fundamental is evidently accompanied by a harmonic of the seventh order, while in other cases the frequency of the overtone is three, four, or five times that of the fundamental. It is clear, also, that the overtones are always harmonics; a fact which has indeed been doubted, but which scarcely requires experimental verification. In some cases, however, peculiarities in the form assumed by the flame lead to considerable uncertainty as to the order of the principal harmonic. Examples of such cases are seen in Figs. 3 and 6. If the indistinct form of the image is due to an improper adjustment of the flame, as in the upper image of Fig. 7, it can easily be remedied. But in several instances the difficulty appears to be inherent in the character of the sound itself, and no adjustment of sensitiveness has resulted in marked improvement.

It will be evident, upon a careful inspection of the figures, that the results cannot in general be explained by the presence of a single overtone. According to the ordinary theory of vowel

formation,[1] we should, in fact, expect that at least two harmonics would usually be present; namely, those which are most nearly identical in pitch with the characteristic tone of the vowel considered. The relative amplitudes of the two will depend upon the amounts by which they differ in pitch from the characteristic, and a complete study of the vowel sounds would require not only a determination of these amplitudes, but also a knowledge of the phases of the various components. The manometric flame is unfortunately not suited for such determinations, for we can feel no certainty that the height of the flame bears a constant ratio to the pressure on the membrane. M. Doumer is indeed of the opinion that the manometric flame may be used to determine differences of phase.[2] His experiments were, however, confined to the case where the only overtone present was the octave of the fundamental; and although the results under these conditions were good, I doubt whether the method would prove satisfactory with more complex sounds.

In the case of vowels which have but a single characteristic tone, the manometric flame may nevertheless be of considerable use. It enables the pitch of the most prominent overtone to be determined in almost all cases, and so indicates which of the harmonics lies nearest to the characteristic tone of the vowel. By a series of experiments in which the same vowel is sung at a variety of different pitches, the ordinary theory may thus be tested, and the characteristic tone determined. With this object in view some fifty negatives were taken when the manometric flame was set in vibration by the vowel *a* as pronounced in the word *father*. The results of these experiments are shown in the table on the following page.

It will be seen that seven different voices have been studied, varying in character from bass to high soprano, while the range of pitches that is represented extends over two and a half octaves. The negatives from which these results are computed were taken at irregular intervals during a period of about two years.

[1] Helmholtz, Tonempfindungen.
[2] Comptes Rendus, 105, p. 222.

THE VOWEL *A* AS IN *FATHER*.

Character of voice.	Pitch of fundamental.	Order of harmonic.	Pitch of harmonic.	Deviation from 736.
(A.) Bass	102	7	714	— 22
" "	196	4	768	+ 32
(B.) Baritone	113*	7	791	+ 55
" "	115*	7	805	+ 69
(C.) Baritone	202	4	808	+ 72
" "	257	3	771	+ 35
(D.) High baritone	125	6	750	+ 14
" " "	127	6	762	+ 26
" " "	128	6	768	+ 32
" " "	131	6	786	+ 50
" " "	135	6	810	+ 74
" " "	148*	5	740	+ 4
" " "	148*	5	740	+ 4
" " "	150	5	750	+ 14
" " "	150*	5	750	+ 14
" " "	156	5	780	+ 44
" " "	158	5 — 4(?)	790, 632	+ 50, — 104
" " "	165	4	660	— 76
" " "	169	4	676	— 60
" " "	183*	4	732	— 4
" " "	188	4	752	+ 16
" " "	189	4	756	+ 20
" " "	191	4	764	+ 28
" " "	193	4	772	+ 36
" " "	198	4	792	+ 56
" " "	199	4	796	+ 60
" " "	200	4	800	+ 64
" " "	208	4 — 3(?)	832, 624	+ 96, — 112
" " "	217*	3	651	— 85
" " "	226	3	678	— 58
" " "	240	3	720	— 16
" " "	241	3	723	— 13
" " "	247*	3	741	+ 5
" " "	252*	3	756	+ 20
" " "	250	3	750	+ 14
" " "	292	3	876	+ 140
" " "	322	2	644	— 92
" " "	379	2	758	+ 22
(E.) High tenor	201	4	804	+ 68
" " "	222	4	888	+ 152
" " "	263	3	789	+ 53
" " "	295	3	885	+ 149
" " "	333	2	666	— 70
" " "	402	2	804	+ 68
" " "	539	2	1078	+ 342
(F.) Soprano	197	5	985	+ 249
" "	262	3	786	+ 50
" "	267	3	801	+ 65
" "	434	2	868	+ 132
" "	518	2	1036	+ 300
" "	667	1	667	— 69
(G.) A child of eight years	275	3	825	+ 89
" " " " " "	394	2	788	+ 52
" " " " " "	440	2	880	+ 44

* Observations marked with a star are from negatives which were considered especially sharp and distinct.

No attempt was made to sing the vowel at defined pitches, the only object being to obtain as great a variety as possible. It will thus be seen that observations have sometimes been duplicated on different days. The uniform agreement of the results in all such cases affords convincing evidence of the reliability of the method. It should be mentioned in this connection that the possibility of selective resonance in the apparatus was tested in a variety of ways. Two manometric capsules, differing widely in size and proportions, were found to give identical results, while changes in the pressure of the gas, and in the burner itself, were also without effect. Although the distinctness of the images was considerably altered by changes in the adjustment of the flame, the essential features remained in all cases the same.

A consideration of the results in the case of the voices marked *A*, *B*, *C*, and *D* in the table, has led to the adoption of the number 736 as the most probable value of the pitch of the characteristic tone. In the last column of the table will be found in each case the difference between this number and the pitch of the most prominent harmonic. It will be seen that in the observations on these four voices the difference in no case exceeds half the fundamental. In those cases where the order of the harmonic is in doubt, the difference reaches its greatest values, while the results obtained from negatives that were considered especially distinct show a close agreement in pitch between the harmonic and the characteristic tone. The results in the case of these voices are thus seen to be in complete accord with the ordinary theory.

The characteristic tone of the last three voices appears to be higher than that of the first four, although the observations are not sufficient to determine its pitch accurately. Thus with a fundamental whose pitch is 197, the harmonic in the case of the soprano voice is 985, while the baritone voice *D*, with practically the same fundamental, has an overtone whose frequency is only 792. In the observations on the voices *E* and *G* there are also indications of a higher characteristic.

It will be remembered that the results obtained by Helmholtz[1] and Willis in the case of the vowel *a* indicate a characteristic

[1] Tonempfindungen.

whose pitch is about 990, while the pitch assigned by Koenig[1] is 896. The low value obtained from my own observations seems to show that the American pronunciation of this vowel is noticeably different from that customary in European languages. The differences in the results obtained with so common a vowel call attention in a striking manner to the necessity for some means of indicating vowel quality which shall be more definite than the use of a letter of the alphabet.

It may not be out of place to point out a few of the applications of the manometric flame which are rendered possible by the use of photography. M. Doumer has already called attention[2] to the advantages of the manometric flame as a means of measuring pitch. Two flames, set in vibration by the sounds whose pitches are to be compared, may be photographed upon the same plate, and a measurement of the negative leads at once to a determination of the relative frequencies. Although this method is probably not capable of the high degree of refinement that is reached by the use of Lissajous' figures, it has the advantage of being applicable to any vibrating body which produces an audible sound. Used in connection with a standard fork, the manometric flame also forms a simple optical chronograph, by which short-time intervals may be measured with great accuracy. The advantages due to the absence of friction will at once be appreciated.

For use in the lecture-room the form of flame here described is found to be a great improvement over those usually employed. With a height of 10 cm., the flame is still highly sensitive to vocal sounds, and is so brilliant that it may be projected, by the aid of a lens and revolving mirror, so as to be clearly seen by an audience of two hundred. It is by no means necessary to construct a special burner for experiments of this kind, since an ordinary oxygen blast lamp, whose central jet was connected with a manometric capsule, has been found to give excellent results.

[1] Comptes Rendus, 70, p. 931. Also Phil Mag., Series IV., Vol. 45.
[2] Comptes Rendus, 103, p. 340.

Summary.

The more important conclusions that have been reached in the foregoing paper may be summarized as follows: —

1. The actinic effect of the manometric flame may be greatly increased by the use of a burner in which the flame is surrounded by pure oxygen. With this form of burner photographs of the vibrating flame may readily be obtained upon a moving plate.

2. The brilliancy of the flame obtained in this way increases its value in lecture experiments, while its actinic effect enables it to be used for the measurement of small intervals of time, as well as for the determination of pitch and timbre.

3. By the aid of photographs of the manometric flame, the conclusion is reached that the characteristic tone of the vowel *a*, as it occurs in the word *father*, has a frequency which is not far from 736 vibrations per second.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1893.

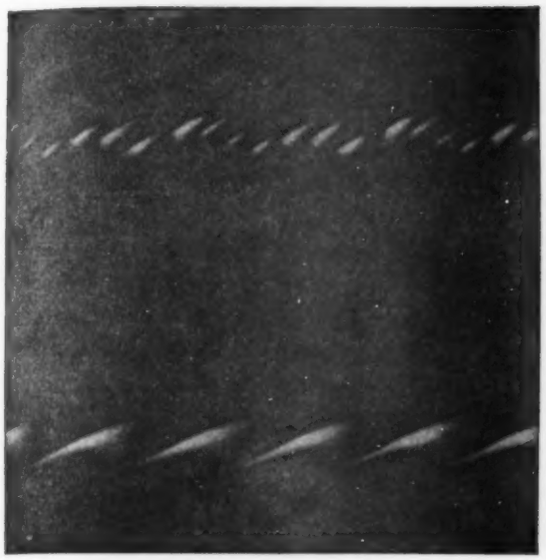

Fig. 1.

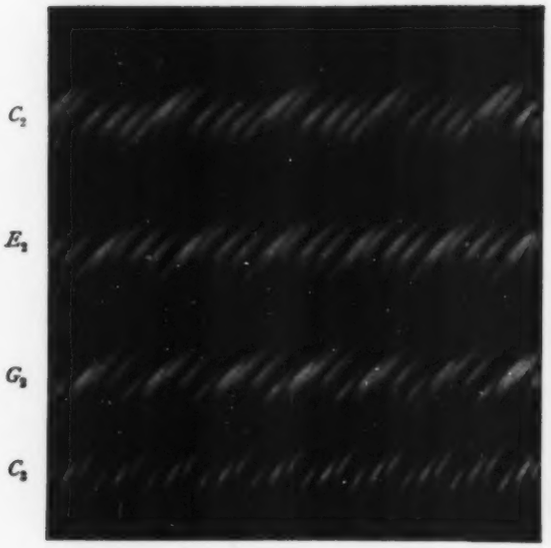

Fig. 3.
a as in *father*.

Fig. 5.
a as in *cat*.

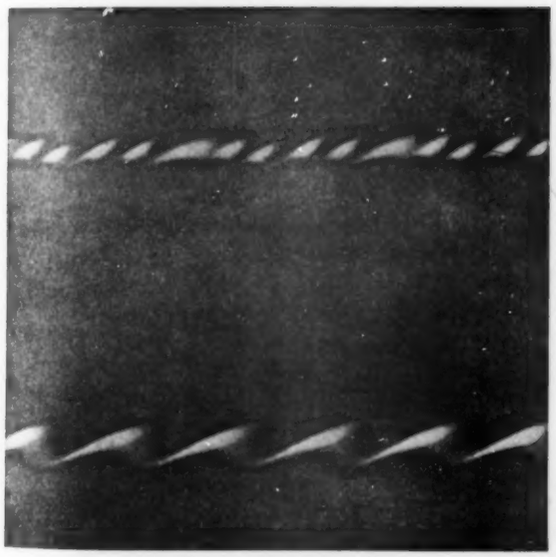

Fig. 2.

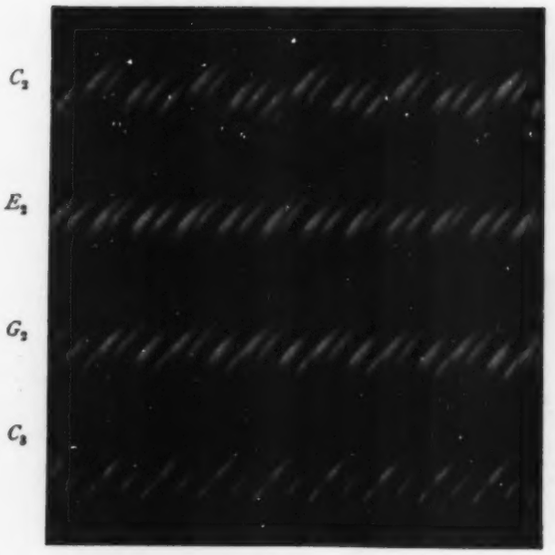

Fig. 4.
a as in *law*.

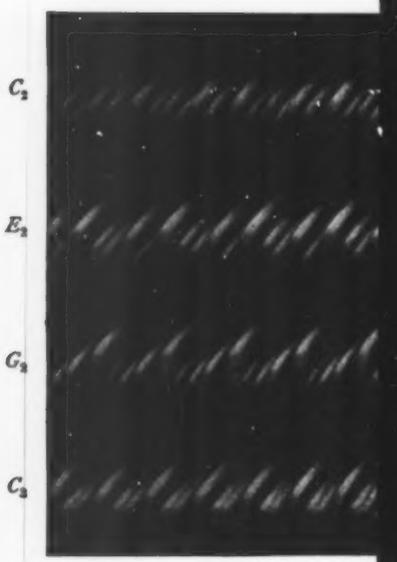

Fig. 6.
o as in *know*.

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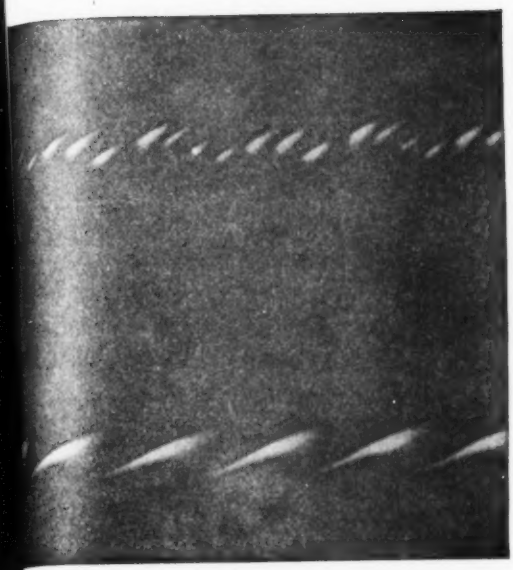


Fig. 1.

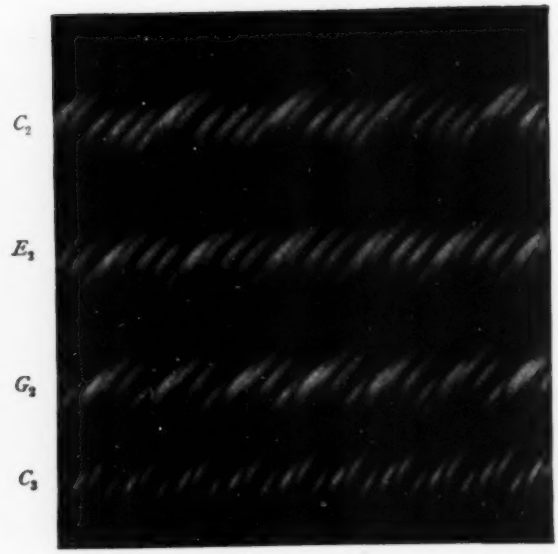


Fig. 3.
a as in *father*.

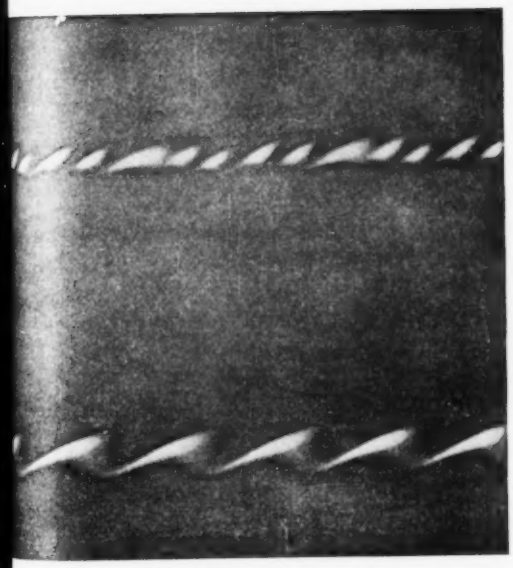


Fig. 2.

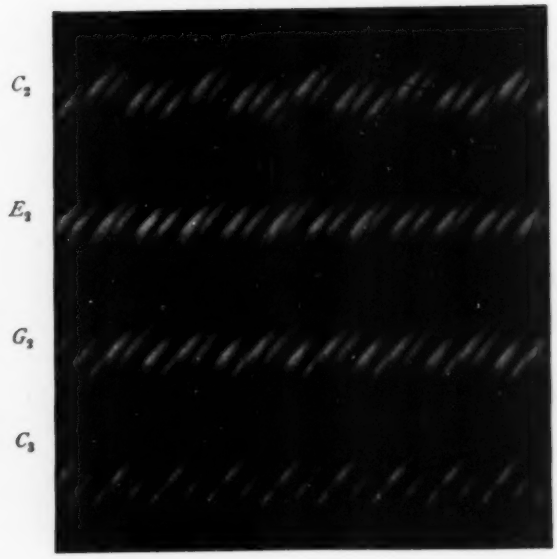


Fig. 4.
a as in *law*.

E.

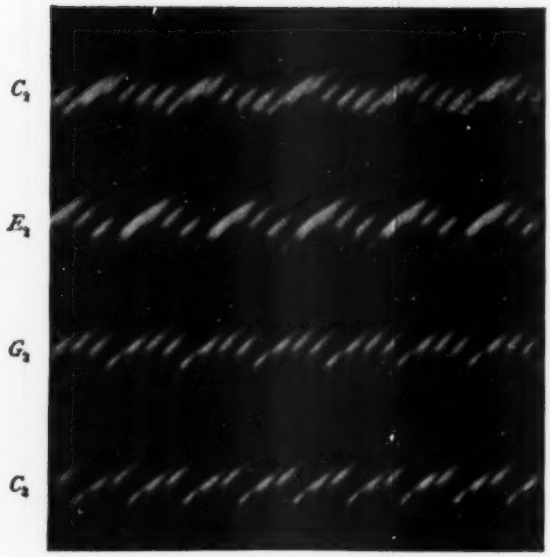


Fig. 5.
a as in *cat*.

Fig. 7.
Four vowels at the pitch 128.

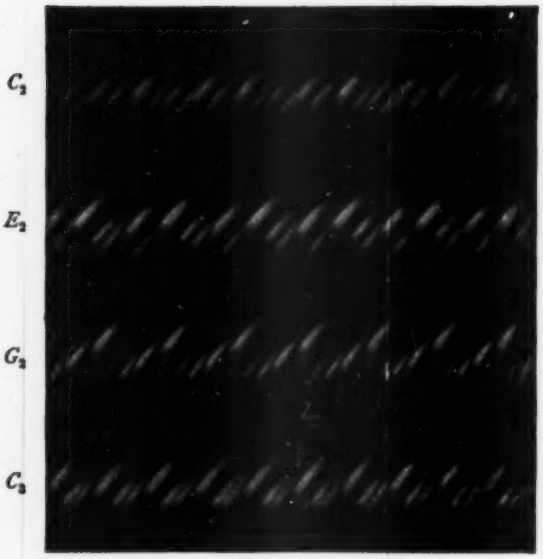


Fig. 6.
o as in *know*.

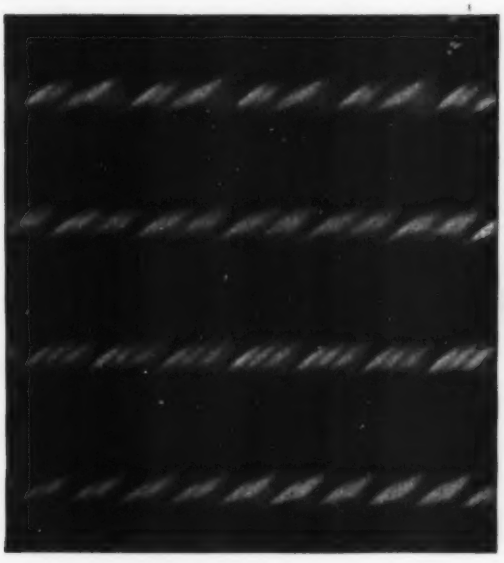


Fig. 8.
M.

E. MERRITT.

GENERAL DISCUSSION OF THE CURRENT FLOW IN TWO MUTUALLY RELATED CIRCUITS CONTAINING CAPACITY.[1]

II.

By Frederick Bedell and Albert C. Crehore.

General Solution for the Currents in Two Mutually Related Circuits with no Condenser.

In symbolic notation (see Johnson's Differential Equations, Chap. V.), equations (26) and (29) may be written

$$(34)\quad i_1 = \frac{1}{D^2 + \frac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \frac{R_1R_2}{L_1L_2 - M^2}} \frac{[R_2 f(t) + L_2 f'(t)]}{L_1L_2 - M^2},$$

and

$$(35)\quad i_2 = \frac{-1}{D^2 + \frac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \frac{R_1R_2}{L_1L_2 - M^2}} \frac{Mf'(t)}{L_1L_2 - M^2}.$$

Resolving the inverse operator into partial fractions, we have the identical equation

$$(36)\quad \frac{1}{D^2 + \frac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \frac{R_1R_2}{L_1L_2 - M^2}} = \frac{L_1L_2 - M^2}{\sqrt{(R_1L_2 + R_2L_1)^2 - 4R_1R_2(L_1L_2 - M^2)}} \left\{ \frac{1}{D + \tau_1} - \frac{1}{D + \tau_2} \right\}.$$

The radical expression which occurs in equation (36) may be written

$$\sqrt{(R_1L_2 + R_2L_1)^2 - 4R_1R_2(L_1L_2 - M^2)} = \sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M^2}.$$

[1] A paper presented before the World's Congress of Electricians, Chicago, Ill., August, 1893.

For simplification, the abbreviations have been used:

$$\tau_1 = \frac{R_1L_2 + R_2L_1 - \sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M^2}}{2(L_1L_2 - M^2)}, \tag{37}$$

and

$$\tau_2 = \frac{R_1L_2 + R_2L_1 + \sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M^2}}{2(L_1L_2 - M^2)}. \tag{38}$$

Placing (36) in (34), we obtain

$$i_1 = \frac{1}{\sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M^2}} \left\{ \frac{R_2f(t) + L_2f'(t)}{D + \tau_1} - \frac{R_2f(t) + L_2f'(t)}{D + \tau_2} \right\}. \tag{40}$$

Similarly (35) becomes

$$i_2 = \frac{M}{\sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M_2}} \left\{ \frac{f'(t)}{D + \tau_1} - \frac{f'(t)}{D + \tau_2} \right\}. \tag{41}$$

Now the linear equation of the first order may be written

$$y = \frac{1}{D + a} f(x),$$

and its solution is known to be

$$y = \epsilon^{-ax} \int \epsilon^{ax} f(x)\,dx + c\epsilon^{-ax}.$$

Hence we have

$$\frac{f(x)}{D + a} = \epsilon^{-ax} \int \epsilon^{-ax} f(x)\,dx + c\epsilon^{-ax}. \tag{42}$$

Replacing $f(x)$ by $R_2f(t)$, and a by τ_1 in this general formula, we have

$$\frac{R_2f(t)}{D + \tau_1} = R_2\epsilon^{-\tau_1 t} \int \epsilon^{\tau_1 t} f(t)\,dt + c\epsilon^{-\tau_1 t}. \tag{43}$$

Similarly we should find

$$\frac{L_2f'(t)}{D + \tau_1} = L_2\epsilon^{-\tau_1 t} \int \epsilon^{\tau_1 t} f'(t)\,dt + c\epsilon^{-\tau_1 t}, \tag{44}$$

and also

$$\frac{Mf'(t)}{D + \tau_1} = M\epsilon^{-\tau_1 t} \int \epsilon^{\tau_1 t} f'(t)\,dt + c\epsilon^{-\tau_1 t}. \tag{45}$$

Substituting these values in (40) and (41), we obtain for the primary current,

(46)

$$i_1=\frac{1}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}\left\{\epsilon^{-\tau_1 t}\int\epsilon^{\tau_1 t}[R_2f(t)+L_2f'(t)]dt\right.$$
$$\left.-\epsilon^{-\tau_2 t}\int\epsilon^{\tau_2 t}[R_2f(t)+L_2f'(t)]dt\right\}+c_1\epsilon^{-\tau_1 t}+c_2\epsilon^{-\tau_2 t};$$

and for the secondary current

(47)

$$i_2=\frac{-M}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}\left\{\epsilon^{-\tau_1 t}\int\epsilon^{\tau_1 t}f'(t)dt-\epsilon^{-\tau_2 t}\int\epsilon^{\tau_2 t}f'(t)dt\right\}$$
$$+c_3\epsilon^{-\tau_1 t}+c_4\epsilon^{-\tau_2 t}.$$

These equations, (46) and (47), are the complete solutions for the current flowing in two mutually related circuits, containing no condenser, with no assumption in regard to magnetic leakage. It is noticed, (37) and (38), that there are two time constants, as in the case of capacity and self-induction in a single circuit; but no oscillatory effect can be obtained in this case, because the expression under the radical is always real for all values of the constants R, M, or L. Further discussion will be deferred until the equations for the other cases are found.

General Solution for Two Mutually Related Circuits in which there is no Condenser, assuming no Magnetic Leakage.

Upon the introduction of the condition of no leak in equations (26) and (29), *i.e.* equating $L_1L_2-M^2$ to zero, they become

(48) $$\left[D+\frac{R_1R_2}{R_1L_2+R_2L_1}\right]i_1=\frac{R_2f(t)+L_2f'(t)}{R_1L_2+R_2L_1},$$

and

(49) $$\left[D+\frac{R_1R_2}{R_1L_2+R_2L_1}\right]i_2=\frac{-Mf'(t)}{R_1L_2+R_2L_1}.$$

The solution of these linear equations of the first order may be written by means of the general formula (42), and we have

$$(50)\quad i_1=\frac{\epsilon^{-\tau t}}{R_1L_2+R_2L_1}\int\epsilon^{\tau t}[R_2f(t)+L_2f'(t)]\,dt+c_1\epsilon^{-\tau t},$$

and

$$(51)\quad i_2=-\frac{M}{R_1L_2+R_2L_1}\epsilon^{-\tau t}\int\epsilon^{\tau t}f'(t)\,dt+c_2\epsilon^{-\tau t},$$

$$(52)\quad \text{where}\qquad \tau=\frac{R_1R_2}{L_1R_2+L_2R_1}.$$

These equations might have been written from the general solutions (46) and (47), but it is easier to derive them independently from the differential equations. It is remarkable how much the consideration of no magnetic leakage simplifies the mathematical expression of the results.

Before entering upon the discussion of these results, it is thought best to obtain the equations for the case in which there is a condenser in one of the circuits, when there is no magnetic leakage.

GENERAL SOLUTION FOR TWO MUTUALLY RELATED CIRCUITS WITH A CONDENSER IN ONE CIRCUIT, ASSUMING NO MAGNETIC LEAKAGE.

The solutions may be obtained from the differential equations (30) to (33), after equating $L_1L_2-M^2$ to zero. With this supposition, when we have the secondary condenser only, the equations become

(53)

$$i_1=\frac{1}{D^2+\dfrac{\left(\dfrac{L_1}{C_2}+R_1R_2\right)}{R_1L_2+R_2L_1}D+\dfrac{R_1}{C_2(R_1L_2+R_2L_1)}}\cdot\frac{\dfrac{1}{C_2}f(t)+R_2f'(t)+L_2f''(t)}{R_1L_2+R_2L_1},$$

(54)

$$i_2=\frac{-1}{D^2+\dfrac{\dfrac{L_1}{C_2}+R_1R_2}{R_1L_2+R_2L_1}D+\dfrac{R_1}{C_2(R_1L_2+R_2L_1)}}\cdot\frac{Mf''(t)}{R_1L_2+R_2L_1}.$$

With the primary condenser only, they become

$$(55)\quad i_1=\frac{1}{D^2+\dfrac{\dfrac{L_2}{C_1}+R_1R_2}{R_1L_2+R_2L_1}D+\dfrac{R_2}{C_1(R_1L_2+R_2L_1)}}\cdot\frac{R_2f'(t)+L_2f''(t)}{R_1L_2+R_2L_1},$$

and

$$(56)\quad i_2=\frac{-1}{D^2+\dfrac{\dfrac{L_2}{C_1}+R_1R_2}{R_1L_2+R_2L_1}D+\dfrac{R_2}{C_1(R_1L_2+R_2L_1)}}\cdot\frac{Mf''(t)}{R_1L_2+R_2L_1}.$$

Resolving the inverse operator in (53) and (54) into partial fractions, we have the identity

$$(57)\quad \frac{1}{D^2+\dfrac{\left(\dfrac{L_1}{C_2}+R_1R_2\right)}{R_1L_2+R_2L_1}D+\dfrac{R_1}{R_1L_2+R_2L_1}}=\frac{R_1L_2+R_2L_1}{\sqrt{\left(\dfrac{L_1}{C_2}+R_1R_2\right)^2-4R_1(R_1L_2+R_2L_1)}}\left\{\frac{1}{D+\tau_1''}-\frac{1}{D+\tau_2''}\right\},$$

where, for abbreviation,

$$(58)\quad \tau_1''=\frac{\dfrac{L_1}{C_2}+R_1R_2+\sqrt{\left(\dfrac{L_1}{C_2}+R_1R_2\right)^2-4R_1(R_1L_2+R_2L_1)}}{2(R_1L_2+R_2L_1)},$$

$$(59)\quad \tau_2''=\frac{\dfrac{L_1}{C_2}+R_1R_2-\sqrt{\left(\dfrac{L_1}{C_2}+R_1R_2\right)^2-4R_1(R_1L_2+R_2L_1)}}{2(R_1L_2+R_2L_1)}.$$

Equations (55) and (56), for the case of the primary condenser only, may be similarly treated by interchanging R_1, L_1, C_1, τ_1', τ_2', with R_2, L_2, C_2, τ_2'', τ_2'', where, for the primary condenser, we have the abbreviations

$$(60)\quad \tau_1'=\frac{\dfrac{L_1}{C_2}+R_1R_2+\sqrt{\left(\dfrac{L_2}{C_1}+R_1R_2\right)^2-4R_2(R_1L_2+R_2L_1)}}{2(R_1L_2+R_2L_1)}.$$

$$(61)\quad \tau_2'=\frac{\dfrac{L_2}{C_1}+R_1R_2-\sqrt{\left(\dfrac{L_2}{C_1}+R_1R_2\right)^2-4R_2(R_1L_2+R_2L_1)}}{2(R_1L_2+R_2L_1)}.$$

This method of obtaining the solution which has previously been given, (34) to (47), finally gives the integral equations in the case of one condenser with the consideration of no leak.

For the case in which there is a condenser in the secondary and no leak, the solutions are

$$(62)\quad i_1=\frac{1}{\sqrt{\left(\frac{L_1}{C_2}+R_1R_2\right)^2-4R_1(R_1L_2+R_2L_1)}}\left\{\epsilon^{-\tau_2''t}\int\tau_2''\left[\frac{1}{C_2}f(t)\right.\right.$$

$$\left.\left.+R_2f'(t)+L_2f''(t)\right]dt+\epsilon^{-\tau_1''t}\int\epsilon^{\tau_1''t}\left[\frac{1}{C_2}f(t)+R_2f'(t)+L_2f''(t)\right]dt\right\}$$

$$+c_1\epsilon^{-\tau_1''t}+c_2\epsilon^{-\tau_2''t}.$$

$$(63)\quad i_2=\frac{M}{\sqrt{\left(\frac{L_1}{C_2}+R_1R_2\right)^2-4R_1(R_1L_2+R_2L_1)}}$$

$$\left\{\epsilon^{-\tau''t}\int\epsilon^{\tau_1''t}f''(t)dt-\epsilon^{-\tau_2''t}\int\epsilon^{\tau_2''t}f''(t)dt\right\}+c_3\epsilon^{-\tau_1''t}+c_4\epsilon^{-\tau_2''t}.$$

When there is a condenser in the primary alone, and no leak, the solutions are

$$(64)\quad i_1=\frac{1}{\sqrt{\left(\frac{L_2}{C_1}+R_1R_2\right)^2-4R_2(R_1L_2+R_2L_1)}}$$

$$\left\{\epsilon^{-\tau_2't}\int\epsilon^{\tau_2't}[R_2f'(t)+L_2f''(t)]dt-\epsilon^{-\tau_1't}\int\epsilon^{\tau_1't}[R_2f'(t)+L_2f''(t)]dt\right\}$$

$$+c_5\epsilon^{-\tau_1't}+c_6\epsilon^{-\tau_2't}.$$

$$(65)\quad i_2=\frac{M}{\sqrt{\left(\frac{L_2}{C_1}+R_1R_2\right)^2-4R_2(R_1L_2+R_2L_1)}}$$

$$\left\{\epsilon^{-\tau_1't}\int\epsilon^{\tau_1't}f''(t)dt-\epsilon^{-\tau_2't}\int\epsilon^{\tau_2't}f''(t)dt\right\}+c_7\epsilon^{-\tau_1't}+c_8\epsilon^{-\tau_2't}.$$

We have now obtained the equations for current flow under various conditions in regard to magnetic leakage and the location of condensers. The solutions have thus far been general; that is, there have been no limitations in regard to the nature of the impressed electromotive force, which may be any function whatsoever of the time. In (46) and (47) we have the expressions for current flow when there are no condensers, without limitation

in regard to there being no magnetic leakage. In (50) and (51) we have the same simplified by the assumption of no leakage. Equations (62) and (63) are the solutions in case of no leak and a secondary condenser; and (64) and (65) are the same for a primary condenser. These general solutions will now be interpreted in turn for certain particular impressed electromotive forces, after which the solution in case of two condensers and no assumption as to absence of magnetic leak will be taken up for an harmonic impressed electromotive force.

Discussion of the General Solution for the Currents in two Mutually Related Circuits with no Condenser.

Equations (46) and (47) are the general expressions for the current flowing in two mutually related circuits due to any electromotive force whatsoever impressed upon one of them. Each equation consists of a particular integral and a complimentary function containing two arbitrary constants of integration to be determined according to the imposed conditions. In order to perform the operations indicated in the particular integrals, in which the electromotive force is expressed as $f(t)$, it is necessary to assume the electromotive force to be some particular function of the time.

General case of "make" or "break."

Ordinarily this would be the case in which initially the two currents are zero and a certain electromotive force is suddenly impressed; or, the case in which initially the two currents have certain assigned values, and the electromotive force is suddenly reduced to zero. All the cases of make or break, that is, the introduction or the removal of the electromotive force, may be generally stated thus: the initial conditions are a primary current I' and secondary current I'', due to some primary impressed electromotive force, the exact nature of which is immaterial; this electromotive force is suddenly changed to a certain known electromotive force. The initial or final values of any of these electromotive forces or currents may be zero.

Let us suppose that the final value of the impressed electromotive force is a constant, $e=f(t)=E'$. Substituting $f(t)=E'$, and $f'(t)=0$ in the general equations (46) and (47), and performing the indicated integrations, we obtain for the primary and secondary currents,

$$(66) \qquad i_1 = \frac{E'}{R_1} + c_1\epsilon^{-\gamma_1 t} + c_2\epsilon^{-\gamma_2 t},$$

$$(67) \qquad i_2 = c_3\epsilon^{-\gamma_1 t} + c_4\epsilon^{-\gamma_2 t}.$$

The arbitrary constants of integration in these complementary functions are to be obtained according to the conditions of the problem. Let I stand for the final steady value $\frac{E'}{R_1}$ of the primary current. Counting time from the time of alteration of the primary electromotive force, we have

(68) When $t=0$, $\quad i_1 = I' = I + c_1 + c_2$,
and $\qquad\qquad i_2 = I'' = c_3 + c_4$.

This gives two equations in which there are four unknown arbitrary constants to be determined, and evidently two more equations must be obtained before they can be found. These equations may be formed from the consideration of the quantities of electricity which will flow in the two circuits while the magnetic field is changing, on account of the change in the impressed electromotive force. The number of lines initially threading the primary circuit is L_1I' due to the primary current, plus MI'' lines due to the secondary; finally, when the primary current has the steady value I, the number of lines will be L_1I. The quantity of electricity (in C. G. S. units) which will flow in the primary, *due to the change in the magnetic field*, will be equal to the change in lines divided by the resistance, or

$$Q_1 = \frac{L_1I' + MI'' - L_1I}{R_1}.$$

Similarly, the initial number of lines threading the secondary will be $L_2I'' + MI'$, which will change to the final value MI; whence

$$Q_2 = \frac{L_2I'' + MI' - MI}{R_2}.$$

Now these values of the quantities of electricity, in primary and secondary, may be obtained from the integrals of the current equations. The integrals of (66) and (67) between the limits zero and infinity, will give the quantities of electricity which will flow during an infinite time from the time of changing the impressed electromotive force; thus, from (66),

$$\int_{t=0}^{t=\infty} i_1 dt = It_\infty + \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2}.$$

Now the first term in the second member is the quantity which will flow, due to the final steady current $I = \frac{E'}{R_1}$, and the remaining two terms represent the quantity which will flow, due to the change in the magnetic field, or

$$Q_1 = \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2}.$$

The secondary flow, from (67), is

$$Q_2 = \int_{t=0}^{t=\infty} i_2 dt = \frac{c_3}{\tau_1} + \frac{c_4}{\tau_2}.$$

This gives us the two additional equations containing the unknown arbitrary constants. From these two equations and the two before obtained (68), the four constants are found to be

$$c_1 = \frac{\tau_1(\tau_2 Q_1 + I - I')}{\tau_2 - \tau_1},$$

$$c_2 = \frac{\tau_2(I' - I - \tau_1 Q_1)}{\tau_2 - \tau_1},$$

$$c_3 = \frac{\tau_1(\tau_2 Q_2 - I'')}{\tau_2 - \tau_1},$$

$$c_4 = \frac{\tau_2(I'' - \tau_1 Q_2)}{\tau_2 - \tau_1}.$$

Equations (66) and (67), with these values substituted for the arbitrary constants of integration, give the values of primary and secondary currents at any time after the alteration of the primary impressed electromotive force. The values of τ_1 and τ_2 so depend upon the constants of the circuits that they are always real, and

so the change of the currents from their initial to final values is gradual and non-oscillatory.

The nature of this change in the currents is shown by the typical curves in Fig. 1 representing a case in which the primary current is changed from a value *OA* to a final value *OB*, the secondary current rising from zero to a maximum, and gradually dying away to zero again. In all cases where the primary is either increased or decreased from one steady value to another by make or by break, the secondary current curve would have a

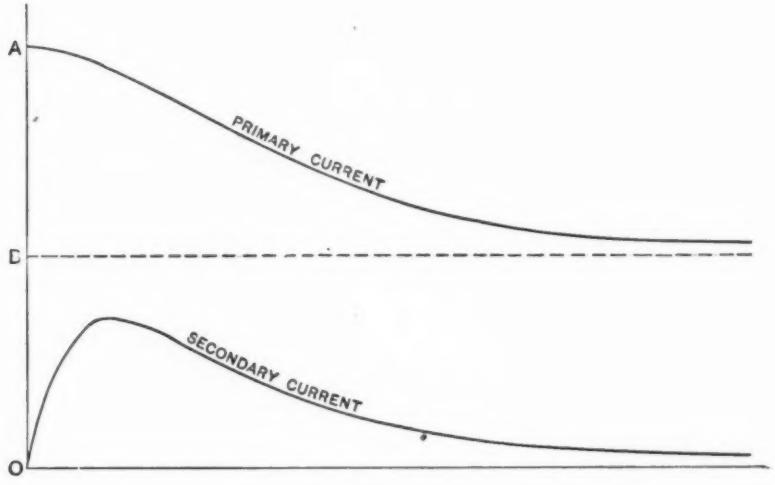


Fig. 1.

shape similar to that shown. The primary current curve as shown is typical for any case of decrease, either to a finite steady value or to zero; if inverted, it would show the change for a corresponding increase.

CASE OF NO MAGNETIC LEAKAGE.

In the hypothetical case of no magnetic leakage the equations take the simpler forms of (50) and (51). According to these equations, the change of the primary and secondary currents from initial to final values, due to a change of the primary impressed electromotive force from its initial to a final steady value, would be represented by exponential curves. Fig. 2 typically represents

such a change. The primary current is changed from a value *OA* to a final steady value *OB* in the opposite direction, the total change being represented by *AB*. The secondary current is represented by an exponential curve with initial value *OC*, and final value zero. Now in the case supposed, the secondary current will be initially zero, and to follow this exponential law it

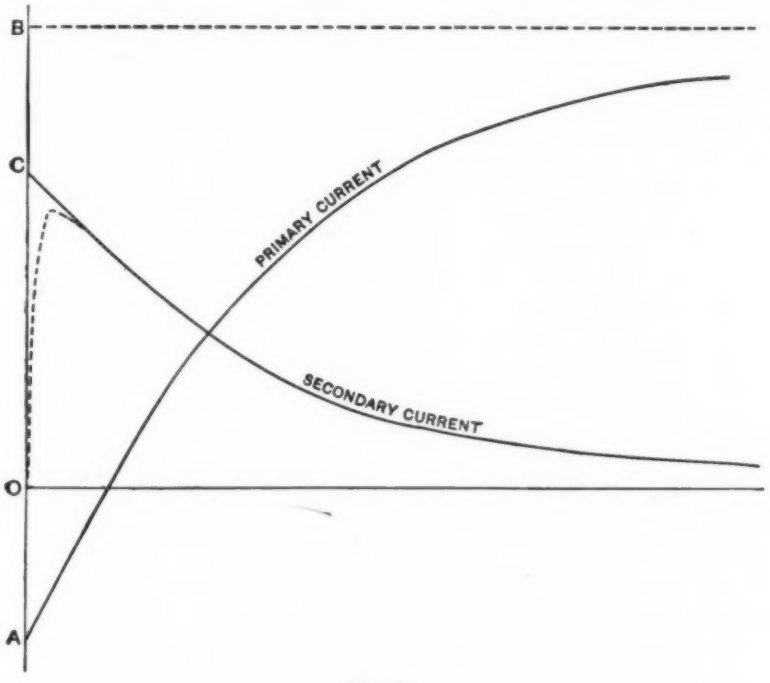


Fig. 2.

would have to immediately assume the value *OC*. Evidently this is impossible, and it shows that the case of absolutely no magnetic leakage is hypothetical. The dotted line shows the nature of the rise from zero, giving a curve as that shown in Fig. 1.

Discussion of Case of Mutually Related Circuits containing a Condenser.

Let us first consider the case in which there is a condenser in the secondary circuit. The general case of make or break will be

treated as before by assuming the primary and secondary currents to be initially I' and I'' respectively, and the impressed electromotive force to be altered to a constant value E'. The expressions for the currents in the primary and secondary at any time t after the change, may be found directly from equations (62) and (63) by substituting $f(t)=E'$, $f'(t)=0$, and $f''(t)=0$. Making these substitutions and performing the integrations indicated, we obtain

$$i_1=\frac{E'}{R_1}+c_1\epsilon^{-\tau_1''t}+c_2\epsilon^{-\tau_2''t}, \tag{69}$$

$$i_2=c_3\epsilon^{-\tau_1''t}+c_4\epsilon^{-\tau_2''t}. \tag{70}$$

These equations are similar to (66) and (67) already discussed, and when the constants τ_1'' and τ_2'' are real, the phenomena attending the make or break do not differ from those already referred to, illustrated in Fig. 1, and need no further explanation.

OSCILLATORY CASE.

The constants of the two circuits may have such values, however, that the constants τ_1'' and τ_2'' are imaginary, in which case the equations (69) and (70) may be transformed, by means of the exponential values of the sine and cosine, into a real form. By referring to the values of the constants τ_1'' and τ_2 given in (58) and (59), we can note whether these values are real or imaginary, and whether or not a transformation is necessary. When $\left(\frac{L_1}{C_2}+R_1R_2\right)^2$ is greater than $4R_1(R_1L_2+R_2L_1)$, the values of τ_1'' and τ_2'' are real, and the equations (69) and (70) may be interpreted as (66) and (67); that is, there is no oscillation. When $\left(\frac{L_1}{C_2}+R_1R_2\right)^2$ is less than $4R_1(R_1L_2+R_2L_1)$, the values of τ_1'' and τ_2'' become imaginary. Equations (69) and (70) can be transformed, however, into the real forms,

$$i_1=\frac{E'}{R_1}+A_1\epsilon^{-\mu t}\sin(\omega t+\Phi_1), \tag{71}$$

$$i_2=A_2\epsilon^{-\mu t}\sin(\omega t+\Phi_2). \tag{72}$$

In the equations A_1, A_2, Φ_1, Φ_2, are constants of integration, which may be determined from the supposed conditions; p and α are constants depending upon the constants of the circuit, thus:

$$p = \frac{\frac{L_1}{C_2} + R_1 R_2}{2(R_1 L_2 + R_2 L_1)}, \tag{73}$$

$$\alpha = \frac{\sqrt{\left(\frac{L_1}{C_2} + R_1 R_2\right)^2 - 4 R_1 (R_1 L_2 + R_2 L_1)}}{2(R_1 L_2 + R_2 L_1)}. \tag{74}$$

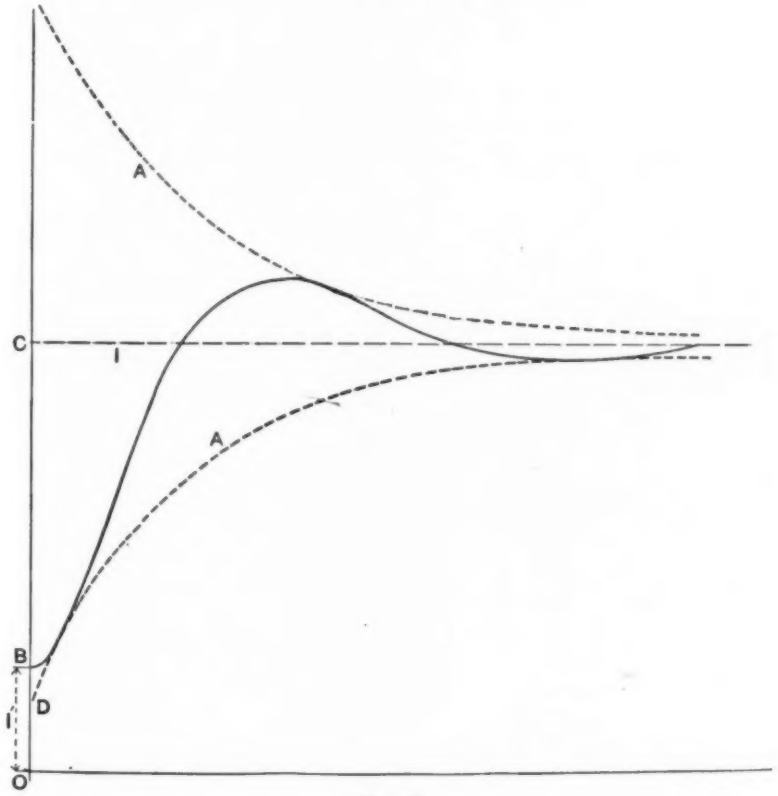


Fig. 3.

Equations (71) and (72) show that both primary and secondary currents will oscillate harmonically about their final values with a period equal to $\frac{2\pi}{a}$, the maximum values of the oscillations

decreasing rapidly with a logarithmic decrement depending upon the value of p. The final steady value of the primary current will be $\frac{E'}{R_1}$, and the secondary will become zero after a short interval of time. The amplitude of the oscillations depends upon the values of A_1 and A_2; their relative phase upon the values of Φ_1 and Φ_2.

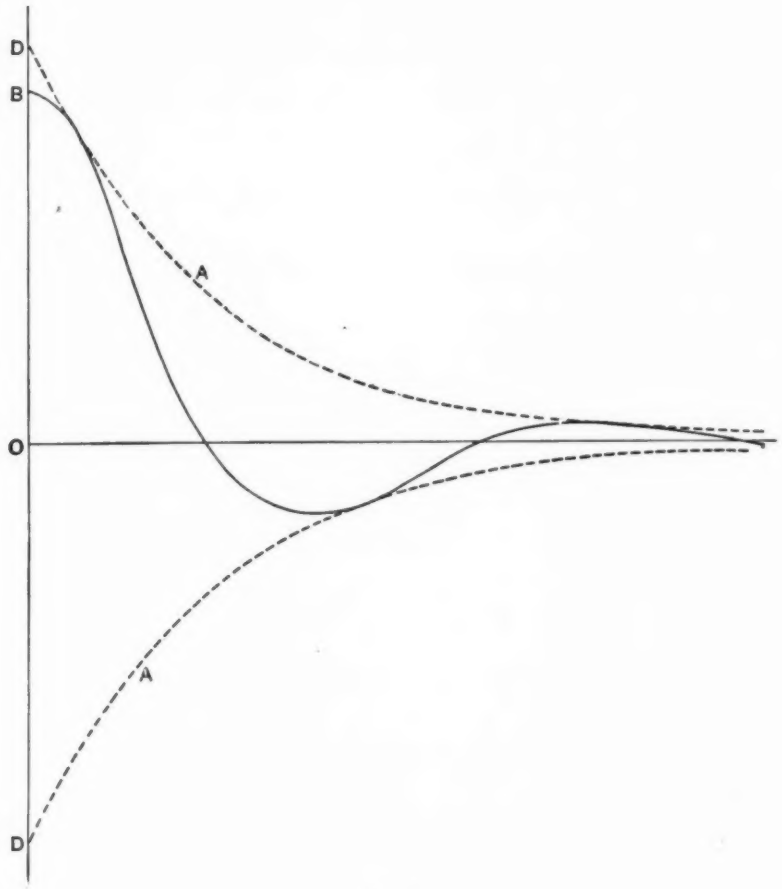


Fig. 4.

The nature of these oscillations will be more clearly understood by inspection of the typical curves in Figs. 3, 4, and 5. Fig. 3 represents the value of the primary current at each point of time

as it changes from an initial value I' to a final value I. The distance BC represents the difference between the initial and final currents. The curve lies between two logarithmic envelopes AA, the initial value of which is equal to A_1 in equation (71), the rate of decay depending upon p. The relation of I' and I to the origin is immaterial; either may be the greater or may be zero.

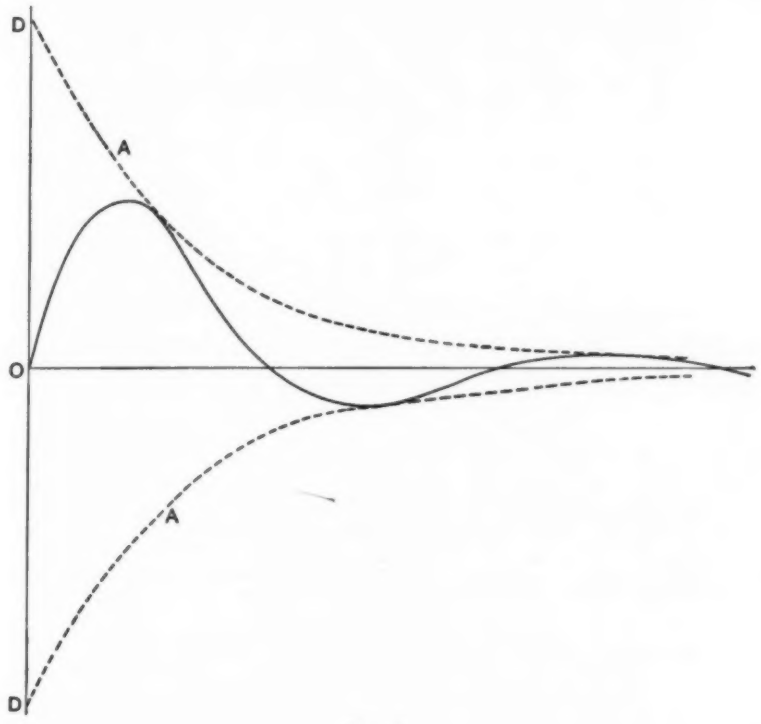


Fig. 5.

If the initial value I' is zero, the origin should be moved from O to B. If the initial value is not zero and the final value is zero, the oscillations are as shown in Fig. 4. Where both initial and final values are zero, the curve in Fig. 5 shows the instantaneous values of the primary current.

The oscillations in the secondary circuit are similar in general character to those in the primary. The nature of the oscillations in the secondary current, as it changes from an initial value OB

to a final steady value, is shown by Fig. 4. Fig. 5 shows the same with the secondary current initially zero.

The foregoing explanation of the oscillations caused by make or break, when the secondary circuit contains a condenser, is simply the interpretation of equations (62) and (63). If the condenser be placed in the primary circuit instead of the secondary, the phenomena will in many respects be the same as those just described, inasmuch as the equations (64) and (65) for the currents in this case are similar to (62) and (63) for the secondary condenser, and a detailed discussion is accordingly unnecessary.

HARMONIC IMPRESSED ELECTROMOTIVE FORCE.

PRIMARY CURRENT.

In taking up the discussion of current flow when the electromotive force impressed upon the primary is harmonic, let us return to the most general expressions for the primary and secondary currents, — the differential equations (16) and (19), — and assume the electromotive force to have a value $E \sin \omega t$.

Substituting in (16),

$$f(t) = E \sin \omega t; \qquad f''(t) = -E\omega^2 \sin \omega t;$$
$$f'(t) = E\omega \cos \omega t; \qquad f'''(t) = -E\omega^3 \cos \omega t;$$

the expression for primary current is obtained:

$$(75) \qquad i_1 = \frac{\frac{E\omega}{C_2} \cos \omega t - R_2 E\omega^2 \sin \omega t - L_2 E\omega^3 \cos \omega t}{(L_1L_2 - M^2)D^4 + (R_1L_2 + R_2L_1)D^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + R_1R_2\right)D^2 + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)D + \frac{1}{C_1C_2}}.$$

Now $D \sin \omega t = \omega \cos \omega t$, and $D^2 \sin \omega t = -\omega^2 \sin \omega t$

whence $D^2 = -\omega^2$, and $D^4 = \omega^4$.

The numerator in (75) may be written as one term by the trigonometric formula,

$$(76) \qquad A \sin \theta + B \cos \theta = \sqrt{A^2 + B^2} \sin\left(\theta + \tan^{-1}\frac{B}{A}\right).$$

Combining the numerator in this manner, substituting $-\omega^2$ and ω^4 for D^2 and D^4, and multiplying by D, we may write (75) thus:

$$(77)\quad i_1=\frac{DE\omega^2\sqrt{R_2^2+\left(\frac{1}{C_2\omega}-L_2\omega\right)^2}\sin\left\{\omega t-\tan^{-1}\left(\frac{1}{C_2R_2\omega}-\frac{L_2\omega}{R_2}\right)\right\}}{\alpha D+\omega\beta},$$

where

$$(78)\quad \alpha=\omega^4(L_1L_2-M^2)-\omega^2\left(\frac{L_1}{C_2}+\frac{L_2}{C_1}+R_1R_2\right)+\frac{1}{C_1C_2};$$

$$(79)\quad \beta=\omega^3(R_1L_2+R_2L_1)-\omega\left(\frac{R_1}{C_2}+\frac{R_2}{C_1}\right).$$

To free (77) from the operator D, operate upon the numerator as indicated, and multiply numerator and denominator by $\alpha D-\omega\beta$. Substitute $-\omega^2$ for D^2, and perform the operation D in the numerator. Having now become rid of the operator D, we have the required integral, which expresses the value of the primary current at any time. Thus:

$$(80)\quad i_1=I_1\sin(\omega t+\Phi),$$

where

$$(81)\quad I_1=\frac{E\omega^2\sqrt{R_2^2+\left(\frac{1}{C_2\omega}-L_2\omega\right)^2}}{\sqrt{\alpha^2+\beta^2}},$$

and

$$(82)\quad \Phi=\tan^{-1}\frac{\beta}{\alpha}-\tan^{-1}\left(\frac{1}{C_2R_2\omega}-\frac{L_2\omega}{R_2}\right).$$

Here Φ is the angle between the primary current and the impressed electromotive force.

SECONDARY CURRENT.

The secondary current is similarly obtained by referring to the general equation (19), and substituting $f'''(t)=-E\omega^3\cos\omega t$; $D^2=-\omega^2$; $D^4=\omega^4$. Making these substitutions, and multiplying numerator and denominator by D, we obtain

$$(83)\quad i_2=\frac{DME\omega\cos\omega t}{\alpha D+\omega\beta}.$$

Operating upon the numerator by D, then multiplying both numerator and denominator by $aD-\omega\beta$, and again operating as indicated, we free the equation from the operator D, and obtain the final integral

$$i_2 = I_2 \sin\left(\omega t - 90^\circ + \tan^{-1}\frac{\beta}{\alpha}\right), \tag{84}$$

where α and β stand for the expressions given in (78) and (79), and

$$I_2 = \frac{ME\omega^3}{\sqrt{\alpha^2+\beta^2}}. \tag{85}$$

These equations just obtained for the primary and secondary currents are the general equations for a transformer, subjected to an harmonic impressed electromotive force, when we have a condenser in each circuit, and make no assumption as to the absence of magnetic leakage; but neglect the change in coefficient of self-induction due to the presence of iron. Their complete discussion would be beyond the limits of the present paper. The generality of the equations may be modified by various limitations, — as, for instance, by the omission of one or both condensers, and by the assumption of no magnetic leakage. The subject can be treated in the inverse order, and the more general obtained synthetically from the simpler cases, the results being the same as those obtained analytically. The identification of the results obtained for a certain case from the general solutions, and those obtained for the same case by a process of building up from simpler cases, is not always readily shown. Suffice it to illustrate in the case in which both primary and secondary condensers are omitted and there is no leak; that is, when

$$C_1 = \infty\ ;\quad C_2 = \infty\ ;\quad M^2 = L_1 L_2.$$

The theory of the transformer for this case has been synthetically developed by the writers,[1] and may be identified with the results obtained for this case from the general discussion in this paper.

The general relation between primary and secondary currents (81) and (85) is

$$I_2 = \frac{M\omega I_1}{\sqrt{R_2^2 + \left(\frac{1}{C_2\omega} - L_2\omega\right)^2}}. \tag{86}$$

[1] "Theory of the Transformer," Electrical World, Vol. XXXI., beginning No. 12, March, 1893.

This expression is evidently in accordance with the law that the current in a circuit is equal to the impressed electromotive force divided by the impediment. For the particular case under discussion this becomes

$$I_2 = \frac{M\omega I_1}{\sqrt{R_2^2 + L_2^2\omega^2}}.$$

The general value for the primary current in (81) reduces for the particular case to

$$I_1 = \frac{E\sqrt{R_2^2 + L_2^2\omega^2}}{R_1 R_2 \sqrt{1 + \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right)^2}}. \tag{87}$$

These results are the same as those obtained on page (340) in the series of articles just referred to.

The angular relation between the primary and secondary currents is seen by a comparison of (80) and (84). The secondary current lags behind the primary by an angle of 90° plus an angle whose tangent is $\frac{1}{C_2 R_2 \omega} - \frac{L_2\omega}{R_2}$. If there is no condenser in the secondary, this lag is $-90° - \tan^{-1}\frac{L_2\omega}{R_2}$, which is in accordance with the well-known transformer diagram. A condenser in the secondary might reduce this angle to 90° or make it even less, as is seen from the equations, and may likewise be shown synthetically. The geometrical construction of transformer diagrams may be thus analytically established.

If we consider that there is no magnetic leakage and put $L_1L_2 - M^2 = 0$, and that there are no condensers in circuit, the expressions (78) and (79) for α and β become

$$\alpha = \omega^3(R_1L_1 + R_2L_2),$$

and

$$\beta = -\omega^2 R_1 R_2.$$

Hence

$$\tan^{-1}\frac{\beta}{\alpha} = \tan^{-1} - \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right),$$

and by (82),

$$\Phi = \tan^{-1} - \left(\frac{L_1\omega}{R_1} + \frac{L_2\omega}{R_2}\right) + \tan^{-1}\frac{L_2\omega}{R}.$$

This easily reduces to

$$\Phi = \tan^{-1} \frac{-\frac{L_1\omega}{R_1}}{1 + \frac{L_1L_2\omega^2}{R_1R_2} + \frac{L_2^2\omega^2}{R_2^2}}, \tag{88}$$

which is identified with the result given on page 340 of the articles in the *Electrical World* referred to above.

Although the analytical expressions might be found by means of the general equations (16) and (19), when there is an harmonic impressed electromotive force, to cover all cases which arise when there are condensers present in the circuits of a transformer, yet it will be found that many problems will lend themselves to readier solution by the graphical methods in which diagrams are built up by synthetic processes. It is considered that the above examples, which identify the results independently obtained from differential equations with the diagrams made from other considerations, are sufficient to show that any diagram has its analytical equations, and any equations, properly derived from the harmonic law, have their corresponding geometrical interpretation.

MAKE AND BREAK WITH HARMONIC ELECTROMOTIVE FORCE.

In the discussion of the current flow in the primary and secondary of a transformer subjected to an harmonic electromotive force, the exponential terms which constitute the complementary function have been omitted from the equations, and the currents are simple harmonic functions of the time. These exponential terms modify the current for a short time after the make, but their effects rapidly diminish and become negligible after a fraction of a second. The exponential terms have been discussed in the earlier part of this paper, and the effects there described are to be superimposed upon the simple harmonic flow of current which would take place if they were not present. Whether these terms are oscillatory or not depends, as before, upon the relation between the various constants of th circuits. When the complementary function is oscillatory, the re. ltant current for a short time oscillates about its final sinusoida form, its form depending upon the

relation between the period of the impressed electromotive force and the natural period of the circuit, and upon the time of introduction of the electromotive force. The periods may be such that distinct beats are obtained. These oscillations are the same in nature as those which occur after the make of a single circuit containing resistance, self-induction, and capacity.

Current Flow in a Single Circuit.

The equations for a single circuit containing resistance, self-induction, and capacity are directly derivable from those for a transformer by assuming that the secondary is removed. To find the values for α and β for this case, take out from (78) and (79) the factor $R_1R_2\omega^2$, and let $\frac{L_2\omega}{R_2}=0$; $\frac{M\omega}{R_2}=0$; $\frac{1}{C_2R_2\omega}=0$. For the primary circuit alone, we then obtain the values

$$\alpha = R_1R_2\omega^2; \qquad \beta=\left(\frac{L_1\omega}{R_1}-\frac{1}{C_1R_1\omega}\right)R_1R_2\omega^2.$$

Substituting these values in (80), the expression for primary current becomes

$$(89) \qquad i_1=\frac{E}{\sqrt{R_1^2+\left(\frac{1}{C_1\omega}-L_1\omega\right)^2}}\sin\left\{\omega t+\tan^{-1}\left(\frac{1}{C_1R_1\omega}-\frac{L_1\omega}{R_1}\right)\right\}.$$

This is the value for the current at any time in a simple circuit containing resistance, self-induction, and capacity when subjected to an harmonic electromotive force, and is fully discussed for general and particular cases in the writers' treatise on *Alternating Currents.*

Conclusion.

We have considered two mutually related circuits with constant coefficients of self-induction, thus not taking into consideration the changes in the coefficient of self-induction, which occur at high magnetization, when iron is present due to the hysteresis loss. The analytical work can only be rigorously correct when there is

no iron, and must be looked upon as an approximation, when iron is present, which is justified when a high degree of magnetization is not reached. We have developed the expressions for the current flow in such circuits in general due to any impressed electromotive force, and have illustrated how they may be reduced to simpler forms for particular cases, — as for the case of an ordinary transformer and for a simple circuit. The limits of this paper make it impossible to enter into a full discussion of the analytical results obtained or to take up the many particular cases covered.

ON THE FREEZING-POINTS OF DILUTE SOLUTIONS. I.

By E. H. Loomis.

THE conflicting views in regard to the nature of solution arise largely from the fundamental disagreement in the results which different observers have found for the freezing-points of dilute aqueous solutions. The difficulties in making such determinations are so great that the results have been affected apparently by large errors. The methods which have been employed to overcome these difficulties are so uncertain in practice that different observers, even when using identical methods, have obtained radically different results, and these have furnished the material for the construction of the conflicting theories. While the estimated errors of the principal observers have been placed by themselves as low sometimes as 0°.0005 C., and seldom higher than 0°.005 C., the observed freezing-points of a given solution have differed by many times the latter amount. One may cite, for example, the case of an aqueous sugar solution, in middle concentration, say $\frac{1}{10}$ normal, or about 3.3 per cent. The observed freezing-points, together with the estimated error of the observer, are given in the following table. These results are obtained by interpolation, for the most part within narrow limits. The first determination of Raoult is reduced to numerical value from his curve of results, which is the only published data. To gain a clearer idea of the magnitude of these differences it must be remembered that the total depression of the freezing-point in a $\frac{1}{100}$ normal sugar solution does not exceed 0°.02. One seems warranted in believing that theories which require for their support such a degree of certainty in the freezing-point of a solution, that 0°.001 C. becomes critical, rest

so insecurely upon the present experimental data that they should be held with some little reservation, and at least advocated with no display of temper.

Observer.	Year of Publication.	Freezing-point.	Estimated error.
[1] Raoult, I.	1886	0°.24	0°.01 (*a*)
[2] Arrehnius, I.	1888	0°.210	0°.005 (*b*)
[3] Traube	1891	0°.235	0°.005 (*c*)
[4] Eykman	1891	0°.216	
[5] Arrehnius, II.	1891	0°.204	
[6] Tamman	1891	0°.206	
[7] Pickering	1891	0°.202	0°.0005 (*d*)
[8] Raoult, II.	1892	0°.205	0°.0020(*e*)

Some two years ago, at the suggestion of Professor Dr. W. Hallwachs, I began to work upon the freezing-points of dilute solutions. The apparatus employed was the well-known one of Beckmann in its improved form,[9] which commends itself so highly on account of its great ease of manipulation and large range of application. The results which I obtained, however, showed so little agreement among themselves, that it soon became evident that the desired degree of accuracy was not to be expected with this method. Not the least of the many difficulties which were encountered, and one which made me always suspicious of my own results, was the inability to free the observations from a large element of arbitrariness. The thermometer seemed to be incapable of assuming a fixed position, and when at times it did become stationary, the slightest incautious jar or change in the rate of stirring, or even continued stirring at a uniform rate, would disturb the mercury. Thus a slight increase in the vigor

[1] Ann. de Chim. et de Phys. (VI.) 8, 1886, p. 313. (*a*) same, (V.) 1883, p. 135.
[2] Zeit. Phys. Chem. II., 1888, p. 491. (*b*) same, p. 493.
[3] Ber. Chem. Berichte, 1891, p. 1853. (*c*) same, p. 1854.
[4] Ber. Chem. Berichte, 1891, p. 1783.
[5] Ber. Chem. Berichte, 1891, p. 2255.
[6] Ber. Chem. Berichte, 1891, p. 2287.
[7] Ber. Chem. Berichte, 1891, p. 3328. (*d*) same as [7], 1892, p. 1860.
[8] Comptes Rendus, 1892, p. 268. (*e*) same as [8], 1892, p. 271.
[9] Zeit. Phys. Chem. II., 1888, p. 638.

of the stirring or its uniform continuation is attended by a steady *rise* in the thermometer, which becomes less and less as the stirring is gradually decreased, and finally the mercury becomes stationary at almost any prescribed point at which the stirring is allowed cautiously to cease altogether. Thus the danger that the bias of the observer may affect the results was so great that the method was finally abandoned.

The only apparent way of avoiding this danger is to insure the presence of a large quantity of ice by allowing a large degree of overcooling to precede the freezing; but here, again, a still more serious difficulty presents itself, namely, the change in the concentration of the solution due to the freezing of a portion of the solvent. My experiments convince me that the corrections which thus become necessary are extremely uncertain, and when they amount, as some observers find, to more than 4 per cent. of the total depression, they cannot fail to produce utter confusion in the results.

My experience, however, with the Beckmann apparatus furnished the key to the method which finally developed itself, since it was during this time that it first became evident how marked was the influence of the temperature of the freezing mixture upon a mixture of ice and water obtained by exposing water in any suitable vessel to its action. The temperature of this partially frozen water, especially when the initial overcooling which always precedes the freezing does not exceed 1° C., is always lower, within certain limits, the lower the temperature of the freezing mixture is chosen, reaching, however, a maximum depression when the freezing mixture becomes about $-7°$ C. Beyond this point a surprising fact presents itself; for now the *lower* the temperature of the freezing mixture, the *higher* becomes the observed temperature of the ice and water mixture surrounded by it. The highest temperature is observed when the freezing mixture is reduced to about $-15°$ C. All this is easily understood as soon as one observes that below a certain point, by no means, however, fixed, but rather dependent on the shape of the vessel and manner and rate of stirring, an ice sheath is formed on the wall of the vessel, which thus serves as a shield against the extreme cold of the exte-

rior bath. When this sheath is complete and compact in texture, the protection which it affords to the remaining ice and water mixture becomes complete, and the water assumes its true freezing temperature. Further reduction of the outside temperature beyond the point at which a complete ice sheath is formed has no appreciable influence upon the inner temperature. That this final temperature should be higher than it is when the freezing mixture is only $-3°$ C., for example, is plain, since at this much warmer temperature the stirring prevents the formation of an ice sheath, and the freezing water is exposed to the full action of the cold environment. It is equally plain that any circumstances which facilitate or hinder the formation of the ice sheath affect the course of these various phenomena, but in no way their general nature.

For similar reasons the temperature of freezing water is determined by the extent of the initial overcooling. When this overcooling is little, the quantity of ice which forms as a result is small, and the influence of the freezing mixture thus manifests itself strongly, while, on the other hand, an initial overcooling of more than 1° C. is followed by a rapid formation of a large quantity of ice throughout the water and the freezing mixture ceases to influence sensibly the thermometer.

That the temperature of an ice and water mixture is exceedingly sensitive to the temperature of its surroundings appears, however, more directly from a simple fact which every observer in this field must have noticed; namely, while allowing the ice to melt in the tube of a Beckmann apparatus, the usual practice is to hold it in the air until, with uninterrupted stirring, the ice is about to disappear. A glance at the thermometer in the meantime shows that as the ice has been melting, the thermometer has steadily risen, slowly at first while the quantity of ice is still relatively large, and finally with great rapidity as the ice nears the point of disappearing. We may safely say, when little ice is present, the effect of a cold freezing mixture or the warm air upon a mixture of ice and water is the same in kind and hardly less in degree than when no ice whatever is present.

This high sensitiveness of an ice and water mixture to the temperature of its surroundings is surpassed by that of a partly

frozen solution. The combined action of these disturbing influences has occasioned large errors in the results, and accounts, I think, in no small measure, for many of the great differences between the results of different observers, — differences which not infrequently amount to 40 per cent. I may perhaps venture to suggest one combination of these influences which accounts for the fact that the observed depressions have been commonly too large. Let us take, for example, a $\frac{1}{100}$ normal solution of any substance. To measure the depression of the freezing-point, we must first find the freezing-point of the solvent, — let us suppose water, — and then that of the solution itself. The difference is the desired depression. The freezing-point of the water is most accurately determined by allowing a large degree of overcooling, and thus insuring the formation of a large quantity of ice. As this in no way affects the validity of the observation, and contributes greatly to the ease and certainty in making it, a common practice has been to allow a large degree of overcooling. On the other hand, in taking the freezing-point of the solution, great care is exercised to confine this overcooling within certain narrow limits, so that the corrections which must be applied on account of the change in the concentration may be as small and uniform as possible. What the effect of this method of procedure is, becomes evident from what has already been said in regard to the influence of the freezing mixture, and its relation to the quantity of ice which is allowed to form; the observed freezing-point of the water, owing to the large quantity of ice present, will be approximately the true freezing-point, while the observed freezing-point of the solution will be too *low*, since the quantity of ice is too small to protect the solution from the action of the freezing mixture. The observed depression is thus too large. This error may easily reach a number of thousandths. As the depression for a $\frac{1}{100}$ normal solution must be multiplied by 100 to obtain the *relative* or so-called *molecular depression*, we observe that an error of a thousandth has the magnitude of a *tenth* of a degree on any chosen scale or curve of results. That some such error has played a prominent part in the various deter-

minations seems evident from a most casual glance at the successive results, from the first observations to the most recent. The observed depressions have steadily decreased, in seeming proportion as these errors have been eliminated, exactly as appears in the case of the $\frac{1}{10}$ normal sugar solution already mentioned.

Before entering upon a detailed description of the present method it needs to be observed that the central idea about which it has grown up lies in the fundamental conception of the freezing-point of a liquid. For the sake of clearness, let us confine ourselves to water as a type.

The freezing-point of water may be defined as *that temperature at which ice and water may exist in contact, and no reaction take place between them.* It is thus the temperature of equilibrium in a mixture of ice and water. However objectionable the term "freezing-point of water" may be when used to designate this temperature, the term "melting-point of ice" is certainly no less so, and I have uniformly used the former as less likely to produce confusion. It is to be observed that this state of equilibrium is exceedingly delicate, and that the mixture of ice and water is at its freezing-point only when this equilibrium is maintained. Thus the temperature of an ice and water mixture is higher when the ice is melting than when it is freezing, and the difference, which may be considerable in spite of thorough stirring, increases in proportion as the forces which tend to disturb the equilibrium become greater, and the extent of surface-contact between the ice and water becomes less. If this surface of contact is small, as is the case when little ice is present and massed in relatively large pieces, the temperature of the mixture may be made to vary many tenths of a degree, according as it is exposed to the temperature of the air or that of a freezing mixture. In the former case, the temperature rises much above the freezing-point, since, owing to the little ice-surface exposed, the melting cannot proceed fast enough to use up the large amount of heat which the mixture receives from the air; in the second case, the temperature is, on the contrary, much below the freezing-point, since, for the same reason, the freezing cannot take place fast enough to prevent the overcooling of the water.

The necessity, then, of protecting the liquid whose freezing-point is to be determined from the disturbing influences of outside temperatures is fundamental; and almost from the beginning, even while still using the Beckmann apparatus, I sought to accomplish this by making the freezing-tube much longer, and so remove the solution as far as possible from the air, and by using a secondary freezing mixture with a temperature only a few tenths below the freezing-point of the liquid to be examined, in which the apparatus was placed at the time of making an observation. This practice still remains a prominent feature of the new method.

The Apparatus.

It consists of a thermometer with enclosed scale divided in hundredths of a degree. Its dimensions are such that a hundredth measures about 0.4 mm. This instrument was made according to special order by Herrn R. Fuess, Berlin, and I may take this opportunity of thanking him for the masterly way in which he met all of my requirements. The scale is divided with the greatest precision, and in all my work with it I have observed no deformation of the mercury meniscus, which has given me so much trouble in other instruments of this class, and which appears to indicate an unclean capillary and impure mercury. This thermometer is held firmly by two paraffined corks in a large test-tube (28 × 3 cm.). One cork is fixed to the thermometer and serves as a stopper to the tube, while the other is placed a little way above the level of the liquid in the tube, and serves to guide the stirrer and maintain the thermometer in a central position. This test-tube has a simple modification which is fundamental to the success of the method. The bottom is bottle-shaped, *i.e.* rounding *inward*, and not outward, as is usually the case with test-tubes. This tube is set in a second test-tube of slightly larger dimensions so as to allow for a millimeter air space between the tubes. They are held together at their upper ends by a short piece of thin-walled rubber tubing of large calibre, while a small rubber band stretched over the inner tube keeps them apart at the bottom. The walls of the tubes should not exceed 1 mm., and the thermometer should reach to within about 2 cm. of the bottom.

The stirrer is a most important part of the apparatus. Of the many forms which I have used, one alone gives complete satisfaction. This is the common "ring" stirrer, with this essential alteration: around the outside of the platinum ring a stripping from a goose feather is bound with platinum wire in such a way that the edge of the feather border just rubs the wall of the tube. This gentle and uniform friction against the wall of the tube, together with the peculiar shape of the bottom of the tube itself, prevents the troublesome massing of the ice between the bulb of the thermometer and the bottom of the tube, and obviates all tendency of the ice to freeze in a sheath. The glass rod of this stirrer runs through openings in the two corks, which act as guides to its motion and so prevent its striking against the bulb of the thermometer. It may be added that in using this stirrer, no tendency has been observed in the ice to mass itself together and float at the surface.

For the sake of clearness and brevity, this double tube with its thermometer and stirrer will be referred to as the *freezing-tube*.

At the time of an observation, this stands up to its neck in a thick mixture of salt water and finely powdered and sifted ice (snow answers the purpose admirably) in such proportions as to have a temperature of 0°.30 C., below the freezing-point of the liquid to be observed. This mixture is contained in a copper vessel 35 cm. high and 6 cm. wide. This is also provided with a ring stirrer and is closed at its mouth with a flat cork through the opening in which the freezing-tube is set. To render the temperature of this mixture as constant as possible, the copper vessel is placed in a thick jacket of felt. This copper cylinder with its weak freezing mixture and felt jacket will be uniformly referred to as the *protection-bath*. This rests upon the base of an ordinary retort-stand, the vertical rod of which supports the framework of a small electric hammer (*H*) similar to those used in common electric call-bells. The coils of this hammer have a combined resistance of about 10 ohms, and are supplied with the current from two accumulators (in series) of about 2 volts each. The blows of this hammer are thus energetic and are delivered directly upon the upper end of the thermometer. Uniform and

vigorous jarring of the instrument is thus maintained during an observation. In addition, two other baths are employed,—one a salt and ice freezing mixture at $-10°$ C., contained in a glass battery jar about 30 cm. high, which the mixture fills to a depth of about 14 cm., or, more exactly, to such a depth that it reaches up to the level of the solution in the freezing-tube when this is placed in it.

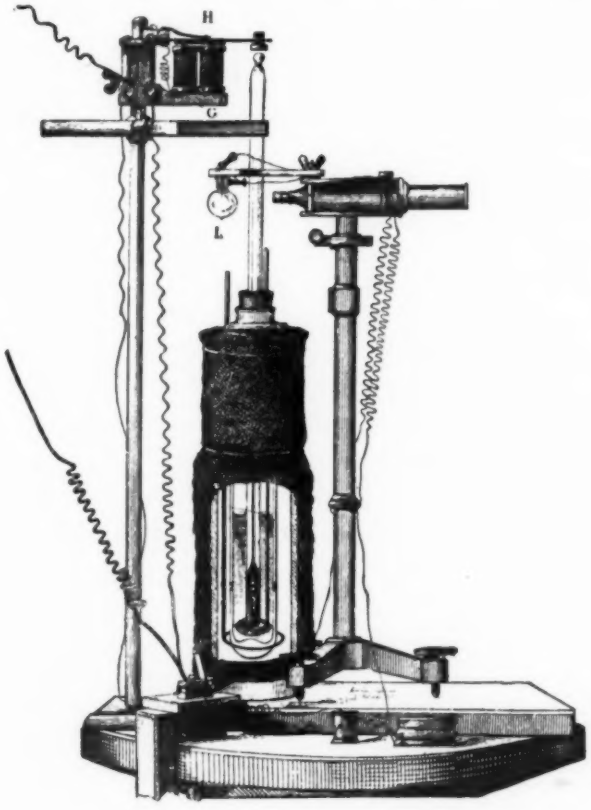


Fig. 1.

This will be known as the *freezing-bath.* The other is merely a mixture of water and coarsely broken ice, in a similar jar, which it fills to such a depth that the freezing-tube may be entirely immersed in it. This is the *melting-bath.* Each of these baths are protected by a heavy wrapping of felt and closed with thick wooden covers, openings in which admit the freezing-tube.

The Reading of the Thermometer.

This is effected with a microscope having a micrometer scale in its eyepiece. The lenses and micrometer are so selected that exactly ten divisions of the micrometer correspond to one scale division of the thermometer. Thus the thousandths of a degree may be directly read and the *ten-thousandths* easily estimated. As the thermometer scale is back of the mercury column, it is obviously impossible to see distinctly the scale and mercury through the microscope at the same time. To read the thermometer, it thus becomes necessary to focus the microscope on the thermometer scale, and bring the micrometer scale into coincidence with it; and then, by a slight sliding backwards of the entire microscope stand, bring the meniscus into focus, in which position the thermometer may be read off on the micrometer scale. To avoid the errors which might possibly arise from this necessary disturbance of the microscope, the stand was made massive and the levelling-screws which supported it rested upon a pane of plate glass which was itself firmly cemented in place. Whatever inconvenience is occasioned by this arrangement is more than compensated for in the fact that, during the interval in which the mercury is coming to its stationary position, no idea may be obtained of what this final position may be, and hence there is no possibility that the observer's bias may affect the result through the unconscious changes in his rate and vigor of stirring, which this bias may induce. To the microscope stand is also attached a small incandescent lamp on the free end of a movable arm, so that it may be readily swung around behind the thermometer, directly in line with the axis of the microscope. The lamp, mercury, and microscope always occupy the same relative position, and errors arising from a variable illumination of the meniscus are avoided. The lamp is only used when the daylight becomes insufficient, and then only during the moment of reading the thermometer. The arrangement of the various parts and the minor details necessary for the few simple adjustments are easily gathered from Fig. 1, taken from a photograph. The figure shows in addition the interior of the protection-bath and freezing-tube.

The Thermometer Corrections.

The thermometer was furnished with a certificate from the German government scientific laboratory at Berlin (Reichsanstalt in Charlottenburg near Berlin), from which the corrections for 0° and −4° — the limits of the scale — were taken. Then by the most careful calibration with a detached mercury column, whose average length in thermometer scale-divisions was 0°.9886, it was found that the interval 0° to 1° was 0°.0015 too great, or instead of being a true degree it was in fact 1°.0015. Owing to the great uniformity in the capillary tube in this region, it was assumed that this error was uniformly distributed over the interval, and thus a correction proportional to the depression had to be added to the observed depression. For the greatest observed depression, namely, 0°.7688 in the case of the $\frac{2}{10}-H_2SO_4$ solution, this correction amounts evidently to 0°.00116. For depressions of less than 0°.1 this correction almost vanishes.

The corrections for variations in the atmospheric pressure are more considerable. The thermometer with its large bulb and minute capillary is a true aneroid barometer, and has a sensitiveness determined by the thickness of the bulb walls and the relations of its other dimensions. In this case, according to the Berlin certificate, a change of 6 mm. in the barometer corresponds to a change of 0°.001 in the reading of the thermometer. Thus this correction may easily exceed 0°.002. To avoid the possible errors which might arise from applying such a large and partly uncertain correction, care was exercised to make all observations on the very *dilute* solutions immediately before or after the determination of the freezing-point of water.

The Method.

The best idea of the practical details of the method may be obtained from a description of the determination of the freezing-point of water, which must be made daily to eliminate the errors due to changes in the thermometer. Further, to render these changes as little as possible, the thermometer is kept during the entire

period of the observations at about 0°. The freezing-tube is filled to the depth of 10 cm. (about 70 cm^3.) with distilled water of a high degree of purity, which is then cooled to the freezing-point and the thermometer put in place. The freezing-tube is now placed in the protection-bath, whose temperature is previously reduced to about $-0°.32$ by the addition of proper quantities of ice or salt. Here it is allowed to remain twenty minutes to insure the cooling of the entire apparatus to the vicinity of 0°.[1] During this interval the temperature of the protection-bath has generally risen to the required $-0°.30$. The freezing-tube is now removed to the "freezing-bath" ($-10°$ C.), where it is kept, with uninterrupted stirring, until the freezing begins. It is then placed in the "melting-bath" (0° C.), and there, by the mere mechanical energy of vigorous stirring, the ice is brought almost to the point of disappearing.[2] This may be known by observing the thermometer itself. It rises slowly so long as considerable ice remains, and more rapidly as the melting proceeds. When but little ice remains, the rise becomes very rapid. A little experience enables one to judge with great certainty when the ice has been melted to the required degree. In the case of water it is generally when the thermometer has reached $+0°.17$.[3]

At this moment the freezing-tube is placed in the freezing-bath where the temperature rapidly falls, until, after an overcooling of between 0°.10 and 0°.20, freezing begins, and the thermometer rapidly rises. At this instant the freezing-tube is thrust into the protection-bath, the electric hammer set in motion, and after two

[1] This precaution is absolutely necessary, since otherwise the first determinations are too high, owing to the warm walls of the freezing-tube and part of the thermometer above the surface of the liquid. This error may amount to more than 0°.004, and has led some observers *arbitrarily* to reject the first observations. When the source of error is discovered, its elimination becomes easy, and the first observations no longer show any irregularities.

[2] For obvious reasons, then, the melting of the ice is accomplished in the "melting-bath" instead of in the warm air, since the disturbing influences of the air and hands give rise to very considerable errors.

[3] This warming of a liquid and its "ice" above its freezing-point may be denoted by the term *overwarming;* while, on the other hand, the cooling of such a mixture below its freezing-point may be denoted by the term *overcooling.* The fact that this latter term is commonly used to denote the cooling of a liquid alone, below its freezing-point before freezing is induced, can hardly lead to confusion.

minutes, during which the stirring has been vigorously maintained, the thermometer is read. In this interval the mercury uniformly reaches its highest point, and remains stationary so long as the jarring and stirring are maintained. After the stirring and jarring are stopped, the mercury continues perfectly stationary from one to two minutes, and then without exception begins slowly to sink. A jar, however, will cause it to drop suddenly much sooner. To obtain a second reading the freezing-tube is again placed in the melting-bath, and the ice again brought to the point of disappearing, at which moment it is transferred to the freezing-bath, and after freezing has again begun, it is placed as before in the protection-bath, when a second reading is made. This is repeated six to ten times, and the middle of the results taken as the required observation. It must not be forgotten that the stirring is to be maintained continuously, from the time that the freezing-tube comes in the freezing-bath, until the thermometer is to be read. The "strokes" should reach from the bottom of the tube to within a few millimeters of the surface of the liquid. The wooden fork (*G*) may be used as an index to control the height to which the stirrer should be lifted. In case the overcooling exceeds 0.°20, it may be known with certainty that the overwarming in the melting-bath was carried too far, and in the following observations the overwarming should be lessened, say to 0°.15. Should, now, the overcooling fall short of 0°.10, one knows with equal certainty that the overwarming was not carried far enough, and this overwarming must be cautiously increased until the subsequent overcooling is brought within the prescribed limits. This relation between the degree of overwarming in the melting-bath and the overcooling in the freezing-bath is most interesting. The experiments show that the greater the overwarming, the greater will be the following overcooling. Thus, one may effect almost any desired amount of overcooling by properly varying the degree of overwarming. In the case of water, for reasons still unknown to me, the degree of overwarming necessary to confine the overcooling within the prescribed limits (0°.10 – 0°.20) is changeable; sometimes, for example, after having remained for many weeks at 0°.15, it may rise to 0°.22, or perhaps fall as low as 0°.08, and

there remain constant for a longer or shorter time. On the contrary, with a solution, this degree of overwarming is very constant, and by varying it the overcooling may be *controlled* to the hundredth of a degree.[1]

It is not without interest to observe that the overwarming necessary to effect the same degree of overcooling in a solution is less than in the case of water, and decreases uniformly as the concentration is increased and with different rates in solutions of different substances.

Thus in a $\frac{1}{10}$ normal solution it is 0°.06 for sugar, while it is only 0°.04 for sulphuric acid. I have not studied this phenomenon systematically enough to warrant me in proposing any explanation of it. To return to the method, we have still to determine in exactly the same way as already described for water, the freezing-points of the solutions, — always remembering that the temperature of the protection-bath must be successively lowered so that for each solution its temperature is 0°.30 lower than the freezing-point of the solution under observation. The difference between this and the freezing-point of water is the observed depression.

It should be added that the various details of the method were developed during a long series of experiments relative to the freezing-point of water alone. In fact, several thousand determinations of the freezing-point of water were made before the various sources of error were so far discovered and suppressed that such a freezing-point could be determined with absolute certainty to the fraction of a thousandth of a degree; and what was more important, that this extreme accuracy could be attained when only the barest traces of ice were present, — a vital condition with solutions to avoid the increase in the concentration which the presence of much ice involves. To indicate to what extent success was attained in this latter respect, a random selection from my note-book may be made.

[1] It is to be noticed that in case ice is once allowed to form in "lumps," as is not infrequently the case with water before the required degree of overwarming is found, all that has been said about the relation between the degree of overwarming and overcooling has no longer any application, and the overcooling can no longer be made to exceed about 0°.08, and is independent of the amount of overwarming. The troublesome lump must be entirely melted by persistent and vigorous stirring in the melting-bath, before further progress may be made.

No.	Overwarming.	Overcooling.	Quantity of ice.	Reading for freezing-point.
1	+ (?)	− 0°.45	Much ice	+ 0°.0274
2	0°.21	0°.07	Little ice	0°.0273
3	0°.22	0°.05	Very little ice	0°.0283
4	0°.23	0°.065	Little ice	0°.0273
5	0°.23	0°.065	Little ice	0°.0270
6	0°.24	0°.070	Little ice	0°.0274
7	0°.34	0°.100	Little ice	0°.0274
8	0°.60	1°.0	Very much ice	0°.0278
9	0°.60	0°.6	Much ice	0°.0273
10	0°.60	0°.9	Very much ice	0°.0278

Middle of 6 observations, little ice present, + 0.02745.
Middle of 4 observations, much ice present, 0.02758.

It thus appears that the disturbing influences of the freezing mixture and air have been so far eliminated that the actual freezing-point of water, and so too that of a solution, may be determined with the greatest certainty even when only a trace of ice is present. *Not until this had been accomplished were any observations on solutions made.*

Although the observations are thus so largely independent of the quantity of ice present, for the sake of a last degree of accuracy, I have always confined the overcooling in the case of water, as is the necessity in the case of solutions, to the limits, 0°.10 to 0°.20. The conditions are thus identical with water and the solutions.

(*To be concluded.*)

MINOR CONTRIBUTIONS.

SOME RAPID CHANGES OF POTENTIAL, STUDIED BY MEANS OF A CURVE-WRITING VOLTMETER.

BY G. S. MOLER.

AT the general meeting of the American Institute of Electrical Engineers, held in Chicago, on June 6th, 1892, a paper was read describing an instrument called a Dynamo Indicator or Instantaneous Curve-Writing Voltmeter, and some of the curves obtained from a dynamo were shown and explained. When the instrument was designed, it was intended to serve mainly as a means of tracing, during a single revolution of the armature of a dynamo, a curve whose ordinates should represent the differences of potential between the two ends of a single coil on the armature, or between one end of a coil and one of the brushes. The instrument consists of an enclosed voltmeter of the permanent magnet form, having a needle whose rate of oscillation is slightly over a hundred per second. By means of a lever, the pointer attached to the needle can be made to press lightly against a small smoked cylinder, which is about one and one-fourth inches in diameter, and two and one-half inches long. The movement of the pointer is through a short arc whose chord is parallel to the axis of the cylinder, so that when the cylinder is revolving, a curve will be drawn upon it, whose ordinates are parallel to the axis. By rolling the smoked drum across a dampened sheet of paper, the smoke may afterwards be transferred to the paper, showing the curve as a white line against a black ground.

Since writing the above-mentioned paper, the instrument has been used for investigating some other sudden changes of potential besides those produced in the coils of a dynamo armature. It is the purpose of this paper to give some of these, which may seem to be of interest.

First, I shall describe some of the curves obtained while running a separately excited motor by means of a series-wound dynamo. The motor, not being loaded, speeds up until it produces a counter-electromotive force sufficient to cut down the driving current till the series-wound dynamo loses its magnetism, and then has its polarity reversed by the

motor, which it then checks and sets revolving in the opposite direction, when the polarity is again reversed, and so on continuously.

The machine used as a motor (see *a*, Fig. 1) was a small shunt-wound Weston dynamo, which at normal output gives 10 amperes at 60 volts. The field of this, which has a resistance of 58.55 ohms, was put in series with a 120-volt incandescent lamp, and was connected to an Edison dynamo which was running at 124 volts.

The brushes of the motor were connected to the terminals of a Siemens and Halske series-wound dynamo *b*,' which at normal rate gives 40 amperes at 60 volts.

When the field of the motor was excited, and the series-wound dynamo was set running, the armature of the motor began to revolve, first in one

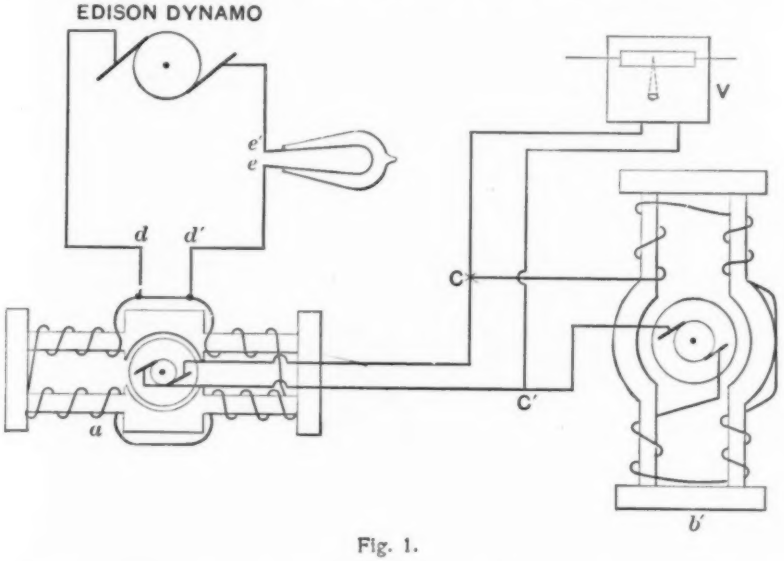


Fig. 1.

direction and then in the other, and so on back and forth. During the experiment it made about eight and one-half revolutions in each direction, which was determined by means of a speed counter. It also, by actual count, reversed direction thirty times in twenty seconds, that is, it ran right-handed fifteen different times, and left-handed just as many more, in twenty seconds. A compass needle, held near one of the poles of the series-dynamo, showed, with each reversal of the motor, a reversal of the polarity of the dynamo, which took place, as near as could be observed, while the motor was at its greatest speed in either direction. An ammeter in the circuit showed a reversal of the current at about the same time. The ammeter was a dead-beat one, and was so quick acting, that readings

could be made with considerable accuracy. The maximum current, which lasted for only a moment each time, was 42 amperes, and the maximum difference of potential of the terminals of the series-dynamo, which also lasted for only a moment, was 41 volts. The incandescent lamp, in circuit with the field, was observed to flash up and be much brighter while the armature of the motor was running in one direction, than while it was running the other. The change in the difference of potential of the terminals of the lamp was found to be from a minimum of 85 to a maximum of 120 volts, while the Edison generator, as said before, was running at a potential of 124 volts. When the motor was not running, the voltmeter around the lamp showed 103 volts, which lies just about half-way between the two readings taken while the motor is in operation. The terminals of the curve-writing voltmeter V were connected at C and C',

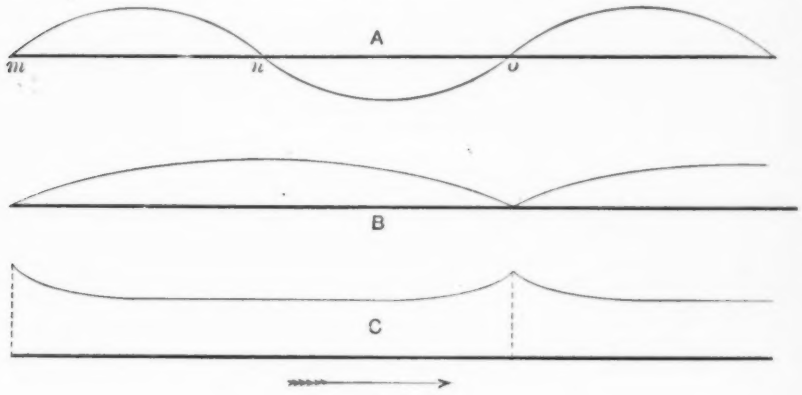


Fig. 2.

Fig. 1, and a curve was obtained, the form of which is shown at A, Fig. 2. In drawing the curves shown in Fig. 2, their ordinates were magnified so as to make them more easily studied. The points m, n, and o, on the curve A, correspond very nearly to the times of greatest motion of the motor, and the ordinates of greatest length, very nearly to the times of reversal of motion when no counter-electromotive force is produced. The curve shows that there are no abrupt changes of potential, for it does not differ greatly from a sine curve. The terminals of the instrument were next connected to the points d and d' to show the changes of potential between the terminals of the field of the motor; this gave the curve B: then the curve C was obtained by connecting to the points e and e' to find the fluctuations at the terminals of the lamp. It was found that at the same time when the curve B drops nearly to the base line, the curve C runs up to a point. This occurs while the potential of the dynamo is

changing most rapidly in one direction but not in the other; that is, there is a high point upon the curve *C* for every pair of reversals. The inductive effect in the field circuit, which caused the lamp to change so much in brightness, was produced by the rapidly changing current in the armature, which was also rapidly changing its motion.

Another study which was made, was of the changes of potential which take place between the carbons of an arc light when it is suddenly started up and while the arc is forming. A Weston lamp, which runs at 20 amperes and 30 volts, was taken, and the instrument was connected to the upper and lower carbons, then the smoked drum was made to revolve at a uniform rate, and the circuit was closed through the lamp. The curve produced is represented at *a*, Fig. 3. At the point *t* the switch is closed,

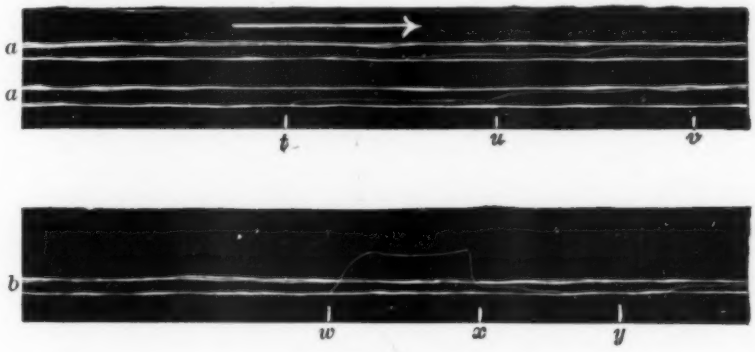


Fig. 3.

and at *u* the arc begins to form: the line from *u* to *v* covers the period during which the carbons are being separated.

Differing greatly from the above curve is the curve *b* obtained from a Thomson-Rice lamp, which operates at 6.5 amperes and 50 volts. In the Weston lamp the carbons touch so long as there is no current flowing, but in the one last mentioned the carbons are held apart by a spring until they are made to touch by the current through the shunt coil of the lamp; then they are separated again, and the arc is formed. There is a cold air gap between the carbons when the circuit is first closed, and the only flow of current that can take place is that through the shunt coil, so the difference of potential between the carbons runs up to some high value, and remains at that till they are made to touch. This is shown by the great height of that part of the curve from *w* to *x*.

This lamp was in series with a rheostat and was connected to an Edison dynamo running at about 125 volts, so the longest ordinates of this part of

the curve must represent that voltage. When the carbons first come together, the contact seems to be rather poor, but as the tips of the carbons heat up, it becomes so good that the ordinates of the curve are practically reduced to zero, as shown at *y*. Immediately after this the carbons are separated, and the curve rises till its ordinates represent about 50 volts, the amount at which this lamp operates.

All of the above-described curves were made over and over again a number of times. They were always of the same form, proving that they were not accidental, but that they represent the actual changes taking place. These are changes which do not manifest themselves in instruments as ordinarily used. Still more delicate and more quickly operating instruments will no doubt develop new facts which will contribute their share toward broadening the field of electrical science.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
July, 1893.

IRREGULARITIES IN ALTERNATE-CURRENT CURVES.[1]

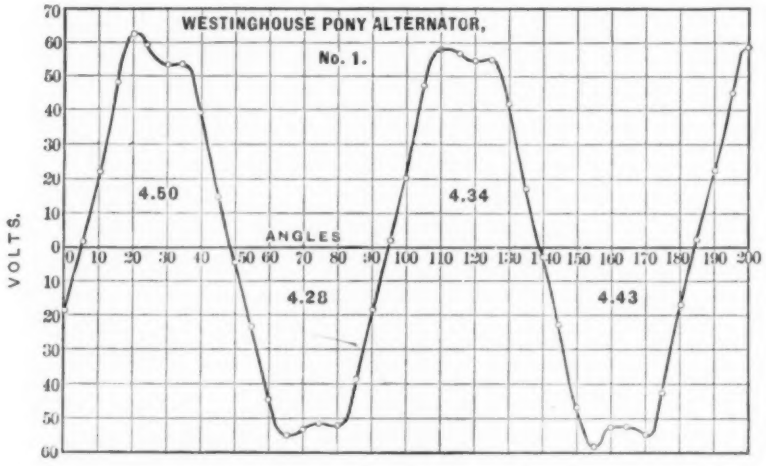
BY FREDERICK BEDELL, K. B. MILLER, AND G. F. WAGNER.

CURVES of a more or less sinusoidal nature are commonly plotted to represent the instantaneous values of an alternating current, and various conclusions are drawn from their forms and relative positions. For any perfectly symmetrical generator the curves for successive periods are the same, both in shape and size, but decided differences may exist between the successive portions of the curves for machines which are not symmetrical. In the larger machines this difference is not likely to exceed one or two per cent., not introducing serious error into the mean results, although occasionally a greater difference may be found. In the smaller machines, however, the instantaneous curves show marked irregularities, so that to take one period as typical is not always justifiable.

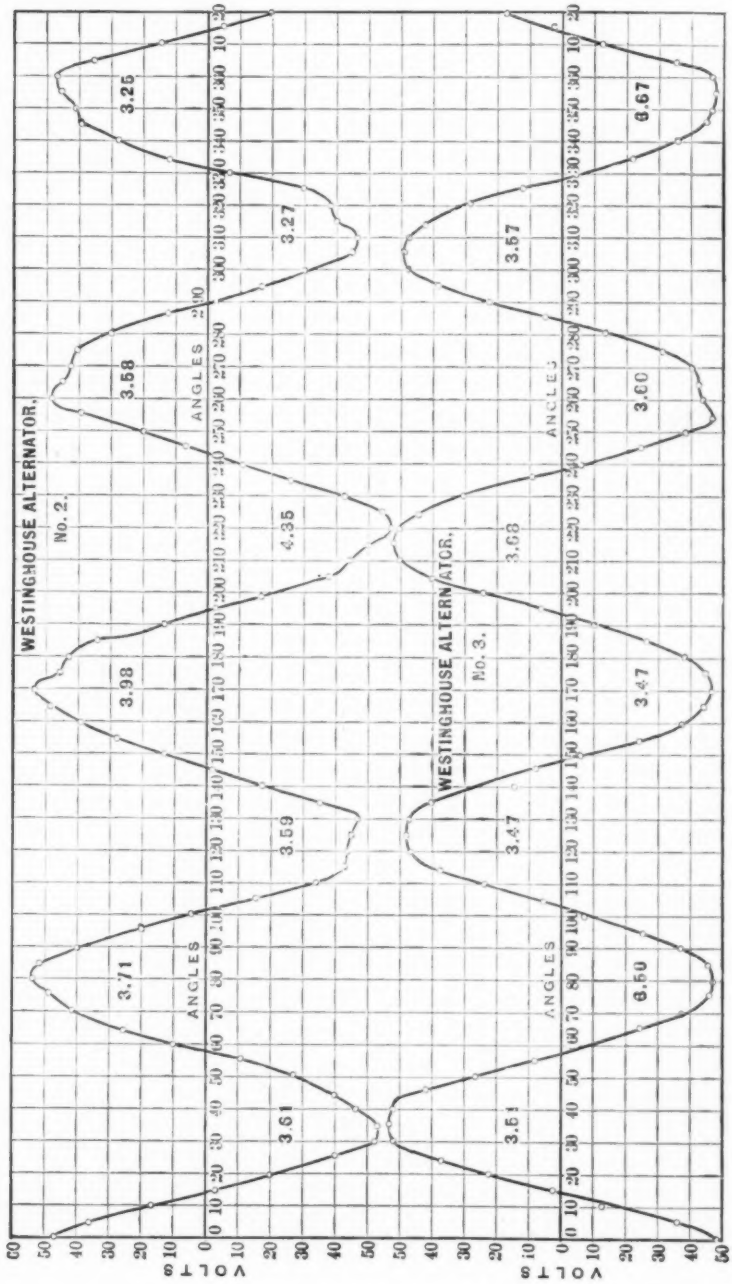
To investigate this point experimentally, curves 1, 2, and 3 were taken from three small eight-pole Westinghouse alternators, showing the instantaneous values of the electromotive force. The electromotive force was measured by means of a Thomson multicellular voltmeter connected around one of two incandescent lamps connected in series across the terminals of the machine, the whole electromotive force being readily found by a proper calibration. The instantaneous reading was obtained by

[1] A paper read before the Madison meeting of the American Association for the Advancement of Science, August, 1893.

means of a revolving contact-maker on the shaft which made the contact at a definite point in the revolution of the armature, as indicated by a graduated disk. The three machines from which curves 1, 2, and 3 were taken were apparently alike in all respects. An inspection of the accompanying figures shows a decided difference in the curves from different machines, and also differences between successive periods of the same machine. This is further seen by comparing the areas of consecutive loops of the curves, as marked upon them. The resistance used was non-inductive, and whether the curves represent current or electromotive force is merely a question of scale. In any case, the ordinates of the current curve represent the instantaneous values of the current, $i = \frac{dQ}{dt}$, whence the



area inclosed by the curve is $\int idt = Q$; that is, the area inclosed by the current curve is proportioned to the quantity of electricity which flows, and for a complete cycle or revolution of the armature must be zero, inasmuch as there is no continual flow one way or the other. Likewise, the ordinates of the electromotive force curve represent the instantaneous values of the electromotive force, $e = \frac{dN}{dt}$, and the area inclosed is proportional to the change in the number of lines of magnetization through the armature. Evidently the algebraic sum of these areas must be zero for a complete cycle, since the magnetization is the same at the beginning and end of a complete revolution of the armature. To establish this point experimentally, curves 2 and 3 were taken, each for a complete revolution of the armature. In these curves, ordinates represent electromotive force, and abscissæ the


WESTINGHOUSE ALTERNATOR, No. 2.
WESTINGHOUSE ALTERNATOR, No. 3.
VOLTS
ANGLES
3.71
3.61
3.98
3.59
3.58
4.35
3.27
3.25
3.51
3.47
3.68
3.47
3.57
3.60
3.50
3.47
3.67

portion of the cycle, with reference to the position of the armature at which readings were taken. The areas of these curves were obtained by a planimeter, and in arbitrary units are: —

Areas, Curve No. 2.		Areas, Curve No. 3.	
Positive.	Negative.	Positive.	Negative.
3.71	3.61	3.51	3.50
3.98	3.59	3.47	3.47
3.58	4.35	3.68	3.60
3.25	3.27	3.57	3.67
14.52	14.82	14.23	14.24
Total, − .30		Total, − .01	

Although, separately, the positive and negative areas widely differ, the sum of the positive and negative areas are very nearly equal, thus experimentally establishing the conclusion reached above.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
July 21, 1893.

NOTE ON THE INFRA-RED SPECTRA OF THE ALKALIES.

B. W. SNOW.

IN the first number of the 48th volume of *Wiedemann's Annalen*, Messrs. Kayser and Runge point out certain errors in my article "Ueber das Ultrarothe Emissionsspectrum der Alkalien," a translation of which appeared in the first and second numbers of the *Physical Review*. These authors then show further that, when certain errors are corrected, my experimental values for the wave-lengths of the infra-red cæsium lines coincide as closely as could be expected with the values predicted by means of their empirical formula. I desire, through the columns of the *Review*, to acknowledge this criticism, and, at the same time, to express to these gentlemen my sincere thanks for their kindness in calling especial attention to these errors, which, however, in the form in which the work was conducted, were unavoidable.

That my values for the wave-lengths of the lines in the infra-red spectra of all the metals studied were not without error I was perfectly aware, and

it was on this account that I gave in detail on page 220 of the original paper, and on page 40 of the translation, the original data from which was compiled the curve of calibration of the prism. Even assuming no constant error to be present in these determinations, these three series of calibrations differ among themselves by amounts of the same magnitude as the errors which Kayser and Runge point out. From the very beginning I saw at once that any method for wave-length determination by means of a calibrated prism is inferior to the beautiful grating method employed by Kayser and Runge. For three reasons, however, under the conditions under which I worked, the use of the prism seemed preferable to that of the grating, notwithstanding the difficulties to be encountered in obtaining a correct curve of calibration: first, because of the greater intensity of the prismatic spectrum; secondly, because of the freedom in this spectrum from all overlapping spectra; and in the third place, because of the failure of the grating in certain spectra, or in certain portions of spectra, to represent the true distribution of energy. The method then employed in the calibration of the prism was the one first used by Becquerel,[1] and afterwards rediscovered by Rubens.[2] While this method is capable of yielding most excellent results, it is quite impossible to obtain by means of it a determination of wave-length with the accuracy attainable by the use of the grating. This failure in the utmost accuracy of this method lies chiefly in determining the exact point of tangency of the curve representing the energy spectrum, crossed by interference-bands, and the outer and inner envelopes of this curve. In my determination of wave-length, therefore, I used this method simply as the best means at hand, and trusted, with the care employed in the calibrations, that the remaining errors might be small.

The other and no less serious criticism made upon this investigation by Messrs. Kayser and Runge refers to the contamination of the various spectra by the presence of impurities in the salts used. This, too, was a difficulty which I recognized throughout the entire series of determinations, and in the original paper I not only made mention of these impurities, but asked that due allowance be made for their presence. With the exception of the sodium chloride, the various metallic salts were obtained from one of the most reliable chemical manufactories in Germany, and a special request was made that they be sent in the purest possible condition. As there was no means at hand for further purification, these salts were used in the condition in which they were received from the factory, although it needed but a glance at the visible spectrum of any of them to detect therein the presence of easily recognized foreign lines.

[1] H. Becquerel, Comptes Rendus, XCVII., p. 71, 1883.

[2] Rubens, Wied. Ann., XLV., p. 238, 1892.

Notwithstanding the fact that each infra-red spectrum undoubtedly contained lines more or less intense which did not belong to the metal there studied, I felt in no way justified in making any corrections or alterations in the distribution of energy as I found it in the infra-red. After passing into the unknown region beyond the limits of the visible spectrum, I did not consider myself capable of saying which lines were true and which were false. Kayser and Runge, however, after a careful analysis of my results, have succeeded in detecting in my cæsium spectrum the spurious lines, and have then shown that when these were eliminated, even with no correction applied to the wave-lengths of those remaining, the latter coincide remarkably well with the values they had previously calculated. Kindly permit me to say in conclusion that this result not only gives me the greatest pleasure, but furnishes one of the strongest proofs of the correctness of their formulæ.

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN,
October 13, 1893.

NOTES.

The American Association for the Advancement of Science. — The forty-second meeting was held at Madison, Wisconsin, August 17–22, 1893. The work of the local committees had been unusually well performed. The various sections found themselves luxuriously housed under a single roof, that of the Science Hall, of the University of Wisconsin, while the state house afforded ample accommodation for large evening gatherings. The Saturday excursion to the "Dells" of the Wisconsin River and the Monday evening lawn party and water fête were features of unusual novelty and beauty.

In Physics, aside from the vice-presidential address "On the Phenomena of the Time-infinitesimal," nineteen papers were read. The list was as follows: —

"Application of Interferential Methods to Measurements of Expansion of Long Bars," Edward W. Morley and Wm. A. Rogers. — "A Preliminary Study of the Constants of the Morley Interferential Comparator," Wm. A. Rogers. — "On Physical Addition or Composition," Alexander Macfarlane. — "Some Rapid Changes of Potential Studied by Means of a Curve-Writing Voltmeter," G. S. Moler. — "Some Applications of Electric Heating in Physical Laboratory Practice," Edward L. Nichols. — "Note on the Use of a Rotating Disk in Photometry," Ervin S. Ferry. — "Irregularities in Alternate Current Curves," F. Bedell, K. B. Miller, and W. F. Wagner. — "Experiments with an Alternate Current Condenser," Frederick Bedell, N. F. Ballantyne, and R. B. Williamson. — "Fatigue in the Elasticity of Stretching," Joseph O. Thompson. — "The Electric Strength of Solid, Liquid, and Gaseous Dielectrics," A. Macfarlane and G. W. Pierce. — "Elastic Properties of Glass," W. S. Franklin and L. B. Spinney. — "An Automatic Toepler Pump," Edward W. Morley. — "An Apparatus for the Generation of Oxygen and Hydrogen by Electrolysis," E. L. Nichols and G. S. Moler. — "On So-Called Negative Lightning," W. LeConte Stevens. — "Note on the Surface Tension of Liquids," Ernest F. Nichols. — "Electrolytic Polarization," John Daniel. — "On the Continuous Spectrum of the Alkalies," B. W. Snow. — "On the Effect of Evaporation upon the Relative Dimensions of Bars of Metal Partially Submerged in Water," Wm. A.

Rogers. — "The Effect of Temperature and of Electric Driving on the Period of Tuning Forks," John S. Shearer.

In the case of the first two papers, increased interest was given to an important topic by the exhibition of the interferential comparator in actual operation. The instrument was mounted in the basement of the physical laboratory of the University and a variety of experiments were performed with it during the meeting.

A number of subjects of interest to physicists were presented in other sections of the Association; notably Vice-President Doolittle's address on "Variations of Latitude," and the papers on "The Atomic Weight of Oxygen," by E. W. Morley; on "An Accurate Method of Measuring Heavy Liquid Pressure," by D. S. Jacobus; "On the Changes in the Dimensions of Metals which may be Due to Changes in Molecular Structure Depending on their Age," by Wm. A. Rogers, and on "The Electro-deposition of Iridium," by W. L. Dudley.

The number of papers on physics was as large as at Rochester (1892) or at Washington (1891), although the total attendance in 1893 was not more than two-thirds of that of the previous year, and was less than half of that of 1891.

The Madison meeting was indeed much the smallest of recent years, the registration falling considerably below that of any meeting of the Association since the Saratoga meeting in 1879. There were several local and temporary causes for the diminished attendance, such as the World's Fair in Chicago, and the great financial depression existing throughout the country, but the failure of the Association to increase with the growth of science in the United States is not to be ascribed to any such influences. There is undoubtedly wide-spread apathy among the men who should be leaders in the cause for the promotion of which the Association was organized. There are, for example, eighteen living past-presidents. Of these one only, the retiring president, who had a definite and important function to perform, was present at Madison. Section B, since its organization in 1882, had been presided over successively by eleven vice-presidents, all of whom are still alive. They were all absent this year. That, under these circumstances, the work in physics (to say nothing of other departments) has steadily improved and that the quasi-science, which it is *not* the purpose of the Association to promote and which in earlier years had been so troublesome an element, has so largely disappeared, is due to the exertions of younger physicists who have entered the Association and whose enthusiasm has not yet had time to grow cold.

In what striking contrast to all this is the condition of the British Association which gathers annually nearly all the foremost men of science of the United Kingdom, together with numerous foreigners of distinction. Its

discussions constitute one of the events of the scientific year, not for England alone, but for the world. Founded, like the younger association on this side of the Atlantic, for the dissemination of science among the people at large, it has yet had time to inaugurate and carry through to completion scientific undertakings of the highest importance. With us, the task which such an association is to perform, must be done under conditions more difficult than those which have existed in Europe; shall the masters stand aside on that account and leave the work to apprentices?

The Electrical Congress. — The International Congress of Electricians (Chicago, August 21–25, 1893) was not the complete success that it might have been had the arrangements been placed in the hands of a permanent, well-organized, and representative body like the American Institute of Electrical Engineers. To compare the meeting with the latest previous event of the kind, we find the registered attendance to be about half as great as at the Electro-technical Congress at Frankfort (1891) and the number of papers read to be in about the same ratio 31 (at Chicago) and 53 (at Frankfort).

The congress suffered, like the numerous other gatherings organized under the official auspices of the World's Fair Auxiliary, in being obliged to meet in the Bedlam of the Chicago Art Institute. The division into sections of pure theory, theory and practice, and pure practice, while entirely logical, did not result in anything approaching a uniform division of the papers to be presented or of hearers. The first and last sections were, relatively speaking, sparsely attended. In spite of these and various other drawbacks, which, under the untoward circumstances, could not be removed even by the untiring efforts of those upon whom the burdens of organization were laid, the Electrical Congress was an important event. Valuable papers were read, and many interesting discussions occurred. Above all, a gathering of electricians and physicists was brought about, the like of which has never before taken place upon the American continent. Professor von Helmholtz was made Honorary President; Dr. Elisha Gray, Chairman; Professor F. B. Crocker, Permanent Secretary; and Professors Rowland, Cross, and Houston, Chairmen of sections A, B, and C. The following is the programme: —

"On the Analytical Treatment of Alternating Currents," Prof. A. Macfarlane. — "Complex Quantities and their Application in Electrical Engineering," Charles P. Steinmetz. — "General Discussion of the Current Flow in Two Mutually Related Circuits containing Capacity," Dr. Frederick Bedell and Dr. Albert C. Crehore. — "Explanation of the Ferranti Phenomenon," Dr. J. Sahulka. — "Measuring the Power of Polyphase Currents," A. Blondel. — "Extended Use of the Name 'Resistance'

in Alternate Current Problems," Prof. W. E. Ayrton, F.R.S. — "Signaling through Space by Means of Electro-Magnetic Vibrations," W. H. Preece, F.R.S. — "Materials for Standards of Resistance and their Construction," Dr. St. Lindeck. — "Variation of P. D. of the Electric Arc with Current, Size of Carbons and Distance Apart," Prof. W. E. Ayrton, F.R.S. — "Ocean Telephony," Dr. Sylvanus P. Thompson, F.R.S. — "Iron for Transformers from the Magnetic Point of View," Prof. J. A. Ewing, F. R. S. — "Note on Photometric Measurement," Prof. B. F. Thomas. — "Some Measurements of the Temperature Variation in the Electrical Resistance of a Sample of Copper," A. E. Kennelly and R. A. Fessenden. — "Various Uses of the Electrostatic Voltmeter," Dr. J. Sahulka. — "On a Method of Governing an Electric Motor for Chronographic Purposes," Prof. A. G. Webster. — "On the Construction of Cables for Subterranean High Tension Circuits," Dr. A. Palaz. — "Periodic Variation of the Candle Power of Alternating Arc Lights," Prof. B. F. Thomas. — "Transformer Diagram Experimentally Determined," Dr. Frederick Bedell. — "London Electrical Engineering Laboratories," Prof. Andrew Jamieson. — "On the Source and Effects of Harmonics in Alternating Circuits," Prof. H. A. Rowland. — "A Pair of Electrostatic Voltmeters," Prof. H. S. Carhart. — "On the Maximum Efficiency of Arc Lamps with Constant Number of Watts," Prof. H. S. Carhart. — "On Direct Current Dynamos of Very High Potential," Prof. F. B. Crocker. — "On an Improved Instrument for Measuring Magnetic Reluctance," A. E. Kennelly. — "The Swinburne-Thompson Unit of Light," Dr. Silvanus P. Thompson, F.R.S. — "Rotary Mercurial Air-Pumps," F. Shulze-Berge. — "The Conversion of Alternating into Continuous Currents," Dr. C. Pollak. — "The Use of Accumulators in Central Stations," Dr. C. Pollak. — "Underground Electric Construction in the United States," Prof. D. C. Jackson. — "A New Incandescent Arc Light," L. B. Marks.

The *chamber of delegates*, which constituted the official portion of the congress, consisted of the following delegates all duly appointed by their respective governments. United States — H. A. Rowland, T. C. Mendenhall, H. S. Carhart, Elihu Thomson, E. L. Nichols. Great Britain — W. H. Preece, W. E. Ayrton, S. P. Thompson, Alexander Siemens. France — E. Mascart, J. Violle, de la Touanne, Hospitalier, Luduc. Germany — H. von Helmholtz, Schraeder, E. Budde, E. Voit, O. Lummer. Italy — Galileo Ferraris. Austria — J. Salhulka. Switzerland — A. Palaz, Thury. Sweden — Wennman. Canada — O. Higman. Mexico — A. M. Chavez.

Professor Rowland was elected president, and Professor Nichols secretary of the chamber.

The principal business transacted by the chamber of delegates consisted in defining the following units, viz. : —

"As a unit of resistance, the international ohm, which is based upon the ohm, equal to 10^9 units of resistance of the C.G.S. system of electro-magnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass of a constant cross-sectional area and of a length of 106.3 centimeters.

"As a unit of current, the international ampere, which is one-tenth of the unit of current of the C.G.S. system of electro-magnetic units, and which is represented sufficiently well for practical use by the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with accompanying specifications,[1] deposits silver at the rate of 0.001118 of a gramme per second.

"As a unit of electromotive force, the international volt, which is the electromotive force that steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of electromotive force between the poles of electrodes of the voltaic cell, known as Clark's cell; at a temperature of 15 degrees C. and prepared in the manner described in the accompanying specification.[1]

"As a unit of quantity, the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one second.

"As a unit of capacity, the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

"As a unit of work the joule, which is equal to 10^7 units of work in the C.G.S. system, and which is represented sufficiently well for practical use by the energy expended in one second by an international ampere in an international ohm.

"As a unit of power the watt, which is equal to 10^7 units of power in the C.G.S. system, and which is represented sufficiently well for practical use by work done at the rate of one joule per second. As a unit of induction the henry is recommended, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt, while the inducing current varies at the rate of one international ampere per second."

The motion to accept the name of Henry for the unit of induction was made by Professor Mascart and seconded by Professor Ayrton. It was unanimously carried.

[1] The specifications referred to are those of the English board of trade, excepting in the case of the Clark cell, the specifications for which are to be modified by a committee consisting of Professors von Helmholtz, Ayrton, and Carhart.

The chamber made no recommendations concerning the rating of arc lamps, and it formulated no definition of a practical standard of light. The very complete system of notation presented by M. Hospitalier was ordered printed, but without official adoption.

Professor von Helmholtz's Visit to America.— The arrival of Baron von Helmholtz in August aroused much enthusiasm among the younger physicists of the country, many of whom have been his pupils. He was accompanied by the Baroness von Helmholtz and by Drs. Lummer, Lindeck, Feussner, Pringsheim, and Leman of the *Electrotechnische Reichsanstalt* in Charlottenburg.

Some Recent Appointments in Physics.— The following are some recent collegiate appointments: Daniel Shea, Ph.D., to the chair of physics in the Illinois State University; Benjamin W. Snow, Ph.D., to the chair of experimental physics in the University of Wisconsin; Frederick Bedell, Ph.D., as assistant professor of physics in Cornell University; John C. Shedd, M.S., to the chair of physics in Marietta College; Arthur L. Foley, A.M., to the chair of physics in Indiana University; Albert C. Crehore, Ph.D., as assistant professor in physics at Dartmouth College.

NEW BOOKS.

Transactions of the Frankfort Congress.[1]

It was remarked by foreign visitors at the electrical exhibition held in Frankfort in 1891, that it was a German rather than an international exhibition, England, France, and the United States being inadequately represented. The same remark might be made, but with less force, with reference to the International Congress for Electro-technics, held during the exhibition. The report of the proceedings of the Congress contains some forty-eight papers, of which all but seven were presented by electricians from the greater Germany, including Austria and German Switzerland. Of the remainder, six were read by English participants in the Congress, while one was from the hand of Professor Hospitalier of Paris.

The proceedings of the Congress are divided into two parts, the first consisting of seven papers read in general session, the other containing the proceedings of the various sections. For the purpose of considering special topics the Congress was divided into sections, to which were assigned: —

First: Theory and Methods of Measurement.
Second: The Technics of Heavy Current.
Third: Signalling, Telegraphy, and Telephony.
Fourth: Electro-chemistry, and Special Applications of the Electric Current.
The Fifth section, for the discussion of legal questions relating to electrotechnics, listened to no papers, but resolved itself into a commission for the consideration of the subject assigned to it.

In giving the first place upon the programme of the general session to Professor Wilhelm Kohlrausch's paper on the proper training of the electrical engineer, "Welches ist der geeigneste Bildungsgang für Elektrotechniker?" the committee of arrangements made a fortunate selection. The published report (pp. 16–39), by the occasional insertion of bracketted remarks, gives some faint indication of the interest with which the paper

[1] Bericht ueber die Verhandlungen des internationalen Elektrotechniker Congresses zu Frankfurt am Main. 2 Parts; pp. xxxix. 141 and 393. Johannes Alt, Frankfurt a/M., 1892.

was received. To appreciate the situation, however, one must have been present in the auditorium at Frankfort and have listened to the discussion which followed.

Kohlrausch, speaking from his experience in the polytechnicum at Hannover, appealed for more mathematics and physics, and chemistry, in the school-training of the electrician. He was met in the discussion by the opinion, already familiar in England and America through the dictum of Kelvin, that electrical engineering is nine-tenths mechanical engineering, and on the part of Slaby of the Charlottenburg technical school, with the statement that all necessary special training in electro-technics might be embraced in a single semester: " . . . ein meist nur einsemestriger Unterricht im Laboratorium," p. 24.

Physicists in America will read these pages with great interest. Few directors of physical laboratories are there in these days who have not been confronted with this question. Many men who are at present entirely occupied in the training of electrical engineers, began like Kohlrausch, "by being physicists," and these at least will sympathize with him in his plea for more thorough study of the fundamental sciences in the schools.

The most striking paper in the first part of these transactions is that of Zipernowsky on interurban electric railways of high speed, in which are outlined with considerable detail the plans for an electric road between Budapesth and Vienna. That a project which contemplates 250 kilometers an hour as an attainable speed should, after two years, still await fulfilment is not surprising. The reader of Zipernowsky's article, however, will certainly be impressed with the practical character of his plans, and will probably admit that he indicates many of the conditions necessary to the solution of the problem of travel at high speeds.

As of special interests to physicists, two subjects in the transactions of the general sessions may be mentioned. Firstly, the discussion of Hospitalier's system of symbols and nomenclature; which is especially useful in that it brings out the fact, since newly emphasized by the treatment which the same excellent scheme received in the chamber of delegates of the Chicago Congress, that the adoption of a uniform system of notation is not a matter of the immediate future; secondly, the discussion following Epstein's paper on *experiment stations for electro-technical investigation* (pp. 116–133).

It is chiefly to the second part of this report, however, that readers interested in physics will turn, and especially to the section devoted to *theory and systems of measurement.*

Here among other matters, chiefly of minor interest, are three contributions from the "Reichsanstalt" in Charlottenburg, viz. "A Study of the Clark and Fleming Normal Elements" by Dr. Lindeck; "Deter-

minations of the Electrical Properties of Copper-Nickel and Copper-Manganese Alloys" by Dr. Feussner, whose results with the latter class (Weston alloys) confirm in the main those previously obtained by B. H. Blood,[1] and of "The Magnetic Behavior of the Alloys of Iron" by Dr. Holborn.

Of much deeper interest than these, from the point of view of the physicist, indeed by far the most important memoir in the volume, is Professor H. F. Weber's paper on "The General Theory of the Glow-Lamp" ("Ueber eine allgemeine Theorie des elektrischen Glühlichtes").

This paper embodies the researches of many years. In it Weber seeks to establish the possibility of computing accurately the absolute temperature of any incandescent carbon filament by means of an equation of the form

$$C \cdot F(T \cdot e^{aT} - T_0 e^{aT_0}) = \frac{I \Delta P}{J},$$

where T is the temperature of the filament, T_0 of the surrounding globe; I is the current, ΔP the difference of potential between the terminals of the lamp, and J the mechanical equivalent of the calorie. F is the radiating surface, a a constant, called the temperature factor, the numerical value of which (0.0043) is the same for all solids. C is termed the constant of total radiation.

The determination of C for thirty-three kinds of glow-lamps showed that filaments are to be classified in two groups, black ($C = 0.000017$) and gray ($C = 0.000013$).

Making use of these constants in the formula, it was found that the "normal" temperature of all filaments is nearly the same; lying between $T = 1565$ (1292° C.) and $T = 1580$ (1307° C.); also that the entire range of temperatures with which one has to do in the case of the glow-lamp is very small; lying between $T = 1400$ and $T = 1650$ as extreme limits.

The value of Weber's results, if they be found capable of direct experimental verification, is not confined to photometry. They offer a foundation upon which to base quantitative study of radiation at temperatures lying between the red heat and the melting-point of platinum such as has not existed hitherto.

The technical electrician will find in the remainder of this volume many matters of the highest interest. Particularly noteworthy is the treatment of the departments of applied electricity in which the Frankfort exhibition excelled; viz. the application of motors and dynamos of the rotary field system to long-distance transmission.

[1] See American Journal of Science, Vol. 39, p. 431.

When we compare the Frankfort Congress, as its results appear in this report, with the Paris Congress of 1881 or that of 1889, we find that it has less claim to be considered a world's congress than these. It was more strictly an *electro-technical* gathering than either, or than the Chicago Congress of 1893, the university element even of Germany, being almost entirely absent. Professors in the technical schools were almost universally present, but the entire group of universities of the German empire furnished but seven participants, out of a total of 472 Germans in attendance, which list did not contain the names of von Helmholtz nor G. Wiedemann nor Heinrich Hertz. But one of the seven university professors (Quincke of Heidelberg) read a paper.

On the other hand, the Congress met at an important period in the development of industrial electricity, and while it burdened itself with no attempts at legislation and offered no official recommendations, it afforded occasion for the presentation of many valuable papers and for much fruitful discussion. The manner in which the work of the compilers has been done is worthy of much praise. The detailed reports of the discussions are faithful to a degree of minuteness not often attained, outside of Germany.

E. L. NICHOLS.

Lehrbuch der allgemeinen Chemie. W. OSTWALD. IIte Auflage; Band II., erster Teil, 1104 pp., Mark 34. Leipzig, W. Engelmann, 1893.

The great advances made in recent years in the theory of chemical processes have made it necessary to rewrite and enlarge Ostwald's famous *Lehrbuch*, which now appears in three volumes. The work is a very clear and complete account of the development of chemical theory, especially from a physical point of view, and includes the quantitative relations constituting the thermodynamical theory of solutions, thermochemistry, electrochemistry and the subject of chemical equilibrium. The historical method of treatment, which was the charm of the earlier edition, is preserved in the new.

An unusual measure of interest is excited by the present volume — on Chemical Energy — for the reason that here for the first time in the history of theoretical chemistry the phenomena of thermochemistry, electrochemistry, etc., are regarded as transformations of the energy forms involved. In the Introduction are emphasized: the substantiality of energy; its forms; its capacity and intensity factors, as

Capacity.	Intensity.	Energy.
distance	× force	= distance energy,
surface	× surface tension	= surface energy,
volume	× pressure	= volume energy,
quantity	× potential	= electricity;

further, the intensity law of Helm (1887), that for equilibrium the intensity of an energy form must be uniform throughout the system; and the *principle of virtual energy changes*, that two energy forms are in equilibrium in a system when their virtual changes are equal. To illustrate this last, suppose a vaporization or similar change, equilibrium being possible between heat and volume energy. The principle requires

$$\frac{\rho}{T}dT = vdp,$$

ρ representing heat of vaporization, etc. This gives

$$\frac{dp}{dT} = \frac{\rho}{Tv},$$

the vaporization formula of Clausius. Again, in a reversible voltaic cell equilibrium may subsist between heat and electrical energy,

$$\frac{Q}{T}dT = edE,$$

$$\frac{Q}{e} = T\frac{dE}{dT},$$

the Gibbs-Helmholtz relation between heat absorption $\frac{Q}{e}$ per unit electrical quantity, and the temperature coefficient of electromotive force. In general, representing capacities and intensities by C and I, energy equilibria are presented by

$$\Sigma CdI = 0.$$

An interesting history of Energetics is appended.

In the chapters on Thermochemistry, Thermodynamics is regarded as that special phase of the energy theory treating the equilibria between heat and work. Here, in addition to the theory, appears a remarkable collection of all reliable thermochemical data.

The conservation of energy, impossibility of creating energy out of nothing ("perpetual motion of the first kind"), and the impossibility of transforming energy without intensity differences, such as gaining work from the constant temperature heat of a mass of water ("perpetual motion of the second kind"), are expressed as the two fundamental

Laws of Energetics.

I. A perpetual motion of the first kind is impossible.

II. A perpetual motion of the second kind is impossible.

The historical account of electrochemistry, from Grotthus to the free ion theory, from Hittorf and Kohlrausch to Arrhenius and Ostwald, from

Volta to Helmholtz and Nernst, is most interesting. The conductivities of electrolytic solutions and the successful application of Ostwald's formula $\alpha^2/(1-\alpha)v = \text{const.}$ are discussed in great detail. With P the electrolytic-solution-pressure of a metal, and p the osmotic pressure of its free ions in a solution of its salt, the potential difference e between metal and solution (*i.e.* the work it does in dissolving) is

$$e = -\int_p^P v dP = -RT\int_p^P \frac{dP}{P} = RT \log \frac{P}{p}.$$

The electromotive force of a reversible cell is then

$$E = RT\left(\log\frac{P_1}{p_1} - \log\frac{P_2}{p_2}\right),$$

where R is the gas-constant, and T absolute temperature. These formulæ are used by Ostwald in the entire construction of an electrochemistry; he draws many new conclusions in doing this and confirms them by experiment; the whole reads like an original paper, and is inspiring in the extreme.

The mutual transformations of radiant energy and chemical energy (photochemistry) complete the volume.

J. E. TREVOR.

The Dynamo: Its Theory, Design, and Manufacture. By C. C. HAWKINS and F. WALLIS. 8vo., pp. xiv + 520. London, Whittaker & Co.

The author's endeavor has been "to furnish to students a simple, yet accurate account of the theory and design of the modern dynamo." "Such a book," they explain in the preface, " . . . must combine both the theoretical and the practical sides; . . . must be free from excessive technicality; and should not be too mathematical; . . . its statements should be entirely accurate."

With this object in view, the writers start with an introductory chapter in which they acquaint the reader with the nature of current flow, which is explained at length by the familiar hydraulic analogies, and they state in a general way the functions of the several members of a dynamo. The reader then becomes familiarized, in Chap. II., with the magnetic field, by the use of the exploring magnet and iron fillings, and obtains a rather clear idea of a line of force from a physical standpoint.

The discussion of the magnetic circuit in the following chapter is, for the most part, clear and sufficient, yet it lacks in certain essential points. A dynamo is fundamentally a magnetic circuit, and no treatise on the dynamo is adequate which is lacking in its treatment of the magnetic circuit. Now the relation which exists between the magnetizing force and the current in the magnetizing coils producing it, is the foundation upon which the

principles of the magnetic circuit are based, and from which every calculation of magnetic field is made. This theorem, the key to all electromagnetic problems, is thus summarily disposed of: "It will be found," (empirically) "that the strength of the field is . . . proportional to the ampere-turns of the solenoid; and . . . inversely proportional to the mean circumference of the ring. If, therefore, the strength of field within the solenoid or number of lines per unit area of cross-section of the ring is symbolized by H, the general law is summed up by the equation

$$H \propto \frac{AT}{l},$$

where AT are the ampere-turns, and l = mean circumference of the ring; or if l is in centimeters, and H in C.G.S. lines per square centimeter,

$$H = k \cdot \frac{AT}{l},$$

where $k = 0.4\pi$, or 1.256.

The total number of lines, Z, passing through the solenoid is equal to the density of the lines per unit area multiplied by the area, or $Z = Ha$, where a = area of cross-section in square centimeters encircled by each loop of the solenoid." The constant "$k = 0.4\pi$, or 1.256," is thrown in without explanation, and the reader is allowed to rack his brains to discover whether it is empirical or found by analysis. The complete proof of the theorem from which this expression is obtained may, perhaps, be beyond the grasp of the average reader for whom the book is intended, but its statement in fine print or a reference to some proper source, whereby the reader may know that the formula has some rational basis, certainly seems called for.

In the following two chapters, on "The Production of an E.M.F.," and "The Magnetic Pull," the two fundamental equations of the dynamo are developed at length. These are

$$(1)\ E = BLV;$$

$$(2)\ F = BLc,$$

where B denotes induction per square centimeter; L, length of conductor; V, its velocity; and c, the current traversing it.

The main portion of the book is devoted to the full description of the various classes of dynamos, typical forms being discussed at length, with one chapter devoted to the detailed description of actual machines, chiefly of English manufacture. Armatures and field-magnets are fully treated and the various methods of compound winding. The idea of self-induction is well brought out in a separate chapter, the development being "simple, yet accurate."

In the discussion of the heating of dynamos, the authors introduce an empirical formula of their own for the maximum rise of temperature of the outside of the armature in degrees Fahrenheit; viz.:

$$t^{\circ}\text{F.} = \frac{85 \times \text{W}}{\text{S}\left(1 + \frac{v}{2000}\right)},$$

where W = total watts expended in armature, S = cooling surface of the armature in square inches, and V = peripheral velocity in feet per minute.

The book closes with a detailed problem in dynamo design, and a chapter on the management and working of dynamos.

The authors have confined themselves closely to their subject, the dynamo, and have well covered the ground which they have assigned themselves. To digress is a temptation, but the writer who starts a chapter with a discussion of 'dry-back and water-tube' boilers and in the same chapter develops the differential equations for alternating-current flow through concentric cables (as seen in a recent technical work), is yielding too much to temptation and presuming too much upon the reader's mental elasticity. Messrs. Hawkins and Wallis keep close to their text and present in a logical way that which directly pertains to the dynamo. They make little claim to originality, — with the exception of their treatment of magnetic leakage, the heating of dynamos, and the E.M.F. of an alternator, — and have simply restated the work of Kapp, Hopkinson, S. P. Thompson, and Ewing; also of Swinburne and Esson. In presenting this material in one volume and systematically developing the subject in a way intelligible to the novice in electrical engineering, the writers have performed a service. Their notation is poor and at times inconsistent. In using the small capitals H and B, they scarcely ascribe the dignity usually accorded these quantities. The use of larger type throughout for symbols would be advantageous. The product of current and electromotive force is variously expressed as *c*E, EC, *ce*, with *ei* and EI at times to avoid monotony. These are minor points, excusable perhaps in a work presenting new material; but in a book of this sort, the merit of which depends entirely upon the style of setting forth, such matters cannot be given too careful attention.

The book is not strong in clear and precise statement; good facts are often concealed amid lengthy explanation. Although the treatment is based upon the simple ideas of current and magnetic field, in no place is the relation given between the volt, the ampere, and the ohm.

The typography is good, the illustrations poor. The book is carefully indexed.

F. BEDELL.

Traité de Photométrie Industrielle. A. PALAZ. pp. vii. + 280. Paris, Georges Carré, 1892.

Professor Palaz's Treatise on Industrial Photometry brings together into convenient form a large amount of useful material which, owing to the industrial prominence of electric lighting, is of especial interest at the present day. In the introductory chapter upon the basis of photometry, in which the law of Bouguer and Lambert is developed, together with brief discussion of the sensitiveness of the eye to various colors, the question of the composition of the light, of artificial illuminants, etc., is taken up. The treatment of the subject of luminosity and of the composition of the light of artificial sources is incomplete and not altogether satisfactory. This is one of the most difficult as it is one of the most important portions of the science of photometry. Upon a knowledge of the relative effects of the various wave lengths of the visible spectrum upon the retina, and of the relative distribution of energy in the spectra of sources to be compared, all exact photometry of lights varying in composition must be based. To give any sufficient account of the subject in the four pages allotted to it by the author was of course impossible. The table representing the visual sensation produced by equal amounts of energy of various wave lengths, according to Langley, is of course of the greatest importance, but to be of use in photometry, it must be supplemented by tables giving the absolute distribution of energy in the spectra of various sources of light. Reliable data upon this latter point are unfortunately not easily obtained. There are, however, tables of relative luminosity (*vide* Vierordt, *Annalen der Physik*, Bd. 137, p. 200; Schumann, *Elektrotechnische Zeitschrift*, Bd. 5, p. 224, etc.), which would have been serviceable to the readers of Professor Palaz's treatise.

The introduction of the results obtained by O. E. Meyer (p. 15) is of doubtful value, since they are not consistent with the measurements of Crova, Pickering, Vogel, and Nichols and Franklin, all of which are in all essential particulars in agreement with one another. In the form in which Meyer's data are presented, moreover, they are misleading. The numerical values given in the table are in point of fact the ratios $\frac{\text{gas-light}}{\text{petroleum-light}}$, $\frac{\text{gas-light}}{\text{sun-light}}$, etc., instead of being, as the context would make it appear, the reciprocals of those ratios. The tables from Violle's measurements (p. 14) are a much more valuable feature of this portion of the book, and the development of methods of determining mean horizontal and mean spherical candle-power, with which the first chapter closes, will be found especially useful. When we come to the description of photometers in the second chapter, we find an excellent summary of the well-known

types, beginning with the original photometer of Lambert, 1760, and coming down to the forms introduced by Lummer and Brodhun, Crova, Weber, and other contemporaries. The portion of this chapter devoted to the spectro-photometer is unfortunately much less complete than that given to the other forms of instrument. Indeed, spectro-photometry in Palaz's work seems scarcely to have received recognition as a legitimate branch of the subject with which his treatise has to do. The third chapter deals with photometric standards, and forms by far the best summary of data concerning them available at the present time. The British and German candles, the Munich candle, and the bougie decimale are here brought side by side for comparison, together with such light sources as the carcel, the petroleum lamp, the acetate of amyl lamp, the Methven slit, and the pentane standard of Vernon-Harcourt. Numerous tables give the results of the many fruitless attempts made by Monie, Dibden, Uppenborn, and others to fix the proper relationship between these various arbitrary standards. The remainder of the book is devoted to a description of auxiliary apparatus, such as the mirror, the revolving wheel, the various forms of holder for getting the distribution of candle power in the case of incandescent and arc lamps, and to the photometry of the incandescent lamp and of the arc lamp. The treatment of these subjects leaves comparatively little to be desired, and it will serve a useful purpose in bringing into juxtaposition material hitherto scattered and difficult of access. Taken all in all, Professor Palaz's treatise is a great advance upon those which have preceded it. It is much more complete than the works of Dibden and of Kruess, and it is written in a broader and more scientific spirit. It will be found a convenient and fairly accurate work of reference for those who have to do with practical photometry, and beyond that will give the reader a glimpse into the field of investigation which this branch of optics presents.

E. L. NICHOLS.

A Guide to Stereochemistry. ARNOLD EILOART. 96 pp. Price $1.00. New York, Alexander Wilson, 1893.

The theory of geometrical isomerism, more briefly termed Stereochemistry, strives to account for the fact that some organic compounds of identical composition and strikingly similar chemical properties yet differ slightly in certain properties, by ascribing to the molecules of these substances different arrangement in space of their constituent atom groups. There has been of late much speculation regarding the stereochemistry of carbon and nitrogen compounds, and it is summarized in the book by Dr. Eiloart, which is designed to serve as a text-book. The author devotes very little attention to the established features of the theory, but discusses the more

recent speculations in considerable detail. For this reason the classical little *Stéréochimie* of van't Hoff (new edition, 1892) is likely to be of more service in giving a clear idea of results actually reached. The latter half of Dr. Eiloart's book is an alphabetical list of literature citations, and to it is appended an illustrated description of wooden models (tetrahedral) designed to represent the "structural formulæ" of organic compounds.

J. E. T.

Experimental Science. By GEORGE M. HOPKINS. 8vo, pp. xiii. + 840. New York, Munn & Co., 1893. 17th edition.

The appearance of the seventeenth edition of this book is evidence of the very large number of persons who find pleasure in the phenomena of physics, particularly when presented in simple guise. The extraordinary popularity of Mr. Hopkins' book is likewise an indication of the prevalence of innate mechanical skill in this country; for a very large proportion of his readers doubtless are not readers merely, but repeat some of the experiments with apparatus of their own constructing.

A work of this kind defies classification. While the traditional sequence of our text-books is followed, it is in no sense a text-book; much less can it lay claim to be a logical well-proportioned treatise. It is entirely devoid of theory and innocent of even the most elementary mathematics; it deals with the most familiar topics of experimental physics, topics many of them already used and abused in a multitude of text-books. Nevertheless, to put this work down as a member of that family, already too large, the "Science made easy," the "A B C of natural philosophy," the "Science of familiar things" family, whose physics is not physics, whose chemistry is not even alchemy, would be a gross injustice. With all its simplicity of statement it is seldom grossly inaccurate; in spite of the plethora of material, which in a book bound to logical continuity of treatment would lead to embarrassment and confusion, the complete independence of each paragraph from all that precedes or follows, makes it as easy to use, thanks to an excellent index, as a dictionary. To the novice, without in any degree supplanting a good text-book, it will be an attractive work, while on the other hand few professional demonstrators of physics will glance over its pages without gleaning therefrom suggestions of new and ingenious methods of illustration.

E. L. NICHOLS.